Mining Gold for Regimes

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Abstract

Regimes are important drivers of returns across financial markets. They are useful not only for ex-post attribution, but also for ex-ante forecasting, and have applications to many types of investors seeking to improve their portfolio's distributional outcomes. In this paper, we propose a new regime identification procedure based on an intuitive and robust underlying methodology, and apply the technique to mine for regimes driving the returns to gold, an asset with a very long history and one with the ability to deliver diversification for portfolios across regimes. The regimes we discover and the underlying methodology we use suggest that a healthy exposure to gold is warranted as part of a strategic asset allocation, especially if we are in a regime in which gold acts as a stable currency or entering one in which it serves as a real asset inflation hedge.

Highlights

- We develop a novel procedure that automatically identifies regimes based on a transparent, flexible, and nonarbitrary underlying methodology.
- We apply the approach to mine for regimes driving the returns to gold, arguably the oldest diversifying asset. The regimes we discover and the long-run historical behavior of gold reveal that it can serve, depending on the macroeconomic environment, either as a real asset proxy, a commodity, and/or a stable currency.
- Our new technique has implications not just for investors' gold allocation specifically, but also for strategic and tactical asset allocation more generally. We present a simple but highly generalizable framework with which allocators could make decisions in the context of historical and forecasted regimes.

Over the last four decades and counting, beginning with papers such as Merton 1980, academics and practitioners alike have devoted considerable energy to estimating time-varying risk premia both in time series and in cross-section. Another way of phrasing the "time-varying" part of that exercise is to suggest that there exist regimes that impact assets across financial markets. Clearly, identification of such regimes would provide investors with, at minimum, the ability to account for the performance and characteristics of global markets.

Perhaps the most salient example of the importance of regimes comes in the form of expansions v. recessions in the business cycle. In the U.S., for example, one common definition used to separate such periods comes from the National Bureau of Economic Research ("NBER")[2]. Looking simply at trailing 1-year returns to the S&P 500 Index during each defined period from March 1977 onwards[3], we find that such returns average 12.3% in expansions and -10.2% in recessions, a 22.5% difference. Clearly, identifying such regimes ex-ante, and even ex-post for attribution purposes, would be very helpful.

Unfortunately, however, the problem with the NBER's definition of regimes is that it is not systematic nor replicable. They note: "There is no fixed rule about what measures contribute information to the process or how they are weighted in our decisions."[4] Prior to 1978, the committee even revised some of its business cycle turning points, and still retains the optionality to do so today. Though no one would dispute that the NBER's recession dates are useful ex-post, our goal is to define a methodology that can be applied systematically both ex-post for attribution purposes and ex-ante for forecasting needs.

In this paper, we develop a novel regime estimation procedure based on the transparent, flexible, and nonarbitrary methodology pioneered by Czasonis, Kritzman, and Turkington 2022a. On the spectrum of more parsimonious to more sophisticated regime identification techniques, ours sits squarely in the middle. We desire a robust approach that nonetheless can capture the complexity and non-linearity inherent in financial market and macroeconomic data, without the downsides of opaque or unstable models that often come with more advanced methodologies.

To illustrate our new framework, we apply it to mine for regimes in the distribution of gold's returns since the 1970s. Gold is a natural asset to select as it is one of the world's oldest diversifying assets and is linked tightly with various macroeconomic variables that may be of interest in a more general set of regimes. Like the three-faced Greek goddess Hecate, or the Hindu triad ("Trimurti") of Brahma, Vishnu, and Shiva, gold's three regime "faces" that we identify provide insights into the performance of an asset that has been around for centuries in investment portfolios. Its "faces" reveal that it can behave, depending on the macroeconomic environment, either as a real asset proxy, a commodity, and/or a stable currency. These are three desirable properties which make gold compelling to include in a portfolio otherwise composed mostly of traditional equity and fixed income risk. This time-varying behavior underscores a key takeaway from Bhansali and Holdom 2021 that we emphasize here: a diversified approach to diversification provides the best chance at improving both short- and long-term portfolio returns and characteristics, especially when one cannot rely on hindsight or data-mining to build robust forward-looking portfolios.

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2. National Bureau of Economic Research. https://www.nber.org/research/business-cycle-dating.

3. Bloomberg. October 31, 2024. This time period matches the start and end of the main data used in our paper.

4. National Bureau of Economic Research. https://www.nber.org/research/business-cycle-dating/business-cycle-dating-procedure-frequently-asked-questions.

This paper is organized into three sections: 1) our new regime identification technique, which builds on the work of Kinlaw et al. 2023; 2) an application to the distribution of gold's returns, with an accompanying discussion of the most salient features of its performance and characteristics across the three gimes we identify; and 3) the implications of our findings for strategic asset allocation in a simplified but highly generalizable framework.

REGIME IDENTIFICATION METHODOLOGY

Recent papers such as Czasonis, Kritzman, and Turkington 2020, Czasonis, Kritzman, and Turkington 2022b, Kinlaw et al. 2023, and Czasonis, Kritzman, and Turkington 2023 have illustrated the potential usefulness of the new concept of relevance in a variety of settings. Relevance suggests that, when it comes to the task of prediction at a particular point in time, maintaining a focus on periods similar to the one of interest but unlike the historical average is paramount. Such observations are more likely to represent useful information for the prediction task relative to data that is either not similar to the current period - thereby potentially indicative of a different regime - or close to the average - potentially representing noise. Mathematically, relevance is defined as shown in equation 1.

$$r_{it} = sim(x_i, x_t) + \frac{1}{2}(info(x_i, \bar{x}) + info(x_t, \bar{x})) \tag{1}$$

In equation 1, $sim(x_i, x_t)$ (similarity) is the Mahalanobis 1936 distance between the current period of interest, x_t, a vector representing the variables selected, and a prior observation of those variables, x_i. $info(x_i, \bar{x})$ (informativeness) is defined similarly as the multivariate distance between each individual observation and the average of the independent variables, $\bar{x}$. Similarity is defined in equation 2, while informativeness is given in equation 3. Note that Σ represents the covariance matrix of the $\boldsymbol{x}$ variables.

$$sim(x_i, x_t) = -\frac{1}{2}(x_i - x_t)\Sigma^{-1}(x_i - x_t)' \tag{2}$$

$$info(x_i, \bar{x}) = (x_i - \bar{x})\Sigma^{-1}(x_i - \bar{x})' \tag{3}$$

Using the definition of relevance in equation 1, we proceed to our regime methodology. In Czasonis, Kritzman, and Turkington 2023, the authors first implement a hidden Markov model to identify regime states, and later apply the concept of relevance to find the Mahalanobis likelihood of each regime at each point in time based on the independent variables they have selected. The result is a remarkably good fit of the regime probabilities to the hidden states despite a completely distinct methodology initially defining those states from that accounting for them. Our approach is different; we maintain an emphasis on the transparency, flexibility, and nonarbitrariness of relevance and solely use it to identify our regimes. Using our own selected variables that are important to, in this case, gold's performance and characteristics, we calculate the relevance of each point in time with all other points in time to recover a T x T matrix somewhat representative of a covariance matrix, in that it encodes the relationship between each time period in our data. From here, we sort the relevance matrix into regimes using k-means clustering, a process that results in an unsurprisingly much tighter fit of the Mahalanobis likelihood of each regime to those identified by the procedure. This goodness of fit indicates that our regimes are determined directly using relevance, and not externally defined prior to our accounting of them.

To sum up in more concrete terms, our regime identification procedure consists of three main stages, underneath which there are several sub-stages/considerations to ensure the robustness of our process:

1. Identify variables of interest.
    - This stage could consist of both quantitative and qualitative inputs, in that variables that are important to, for example, gold's performance and characteristics, could be identified with techniques such as sparse regression and/or specific domain knowledge. We leave this stage to the reader for generalizations.
    - Notably, this is the only stage of our process[5] for which inputs are required; it is otherwise entirely systematic.
2. Calculate the relevance of every data point to all other data points. This yields a T x T relevance-based encoding matrix ("RBEM").
    - The matrix encodes information about the relevance of each point in time within the data to all other points in time.
    - Although somewhat similar to a covariance matrix between each date, the RBEM, while symmetrical, does not have the property that the largest elements are those on the diagonal. The implication is that the most relevant observation to a particular date may not be that date itself, a perhaps surprising result. This is possible because relevance depends not only on similarity - which is zero (most relevant) for an observation relative to itself - but also on informativeness, which from equation 3 shows that how far a set of observations are from their average is important; more noisy observations close to the center of the distribution are less relevant.
    - To the extent that there are a sufficiently large number of observations and that the sample average and covariance matrix are unbiased and consistent estimates of the population (or at least held fixed), adding new data as time progresses does not change the historical relevance between past data points. This makes our approach not only suitable for an historical accounting of regimes, as we use it for in this paper, but also for an out-of-sample forecasting methodology.
3. Identify regimes using k-means clustering on the RBEM data.
    - The algorithm iteratively sorts the data into k clusters by identifying the best cluster centers ("centroids") that minimize the distance between each data point and its respective center on average.

5. The only other input is in selecting the number of regimes, k. In this case we choose three, representing a common-sense balance between parsimony and complexity.

- Since a T x T is a high-dimensional object upon which to fit a model like k-means, we first conduct a principal components analysis ("PCA") to extract the linearly orthogonal features that explain at least 95% of the variance of the RBEM. This makes the task of identifying the appropriate centroids less subject to the curse of dimensionality.
- k-means clustering is not deterministic, so in the sense that it is sensitive to the initialization points, we fit 1,000 iterations of the algorithm with different starting points using a set k value (in this case, 3) and identify the one for which the average L2 norm score between each PCA data point and its respective centroid is minimized. We take the resulting regimes from this minimum to be the "global" best fit for a set k value; though our goal in this paper is explanatory in nature and therefore uses the full sample, such an approach lends itself naturally to out-of-sample regime forecasting using cross-validation.

APPLYING THE METHODOLOGY TO GOLD

To illustrate the process, we first select eight variables important to gold's performance and characteristics. These include two measures of gold valuation, various relevant macroeconomic and financial market indicators, as well as the trailing performance of gold itself. The variables specifically are the:

1. 1-year lagged carry implied by the front two months of gold futures contracts;
2. 1-year lagged level of real gold prices (gold price scaled by the level of headline U.S. CPI, a ratio that can act as a kind of valuation monitor for gold along with the level of negative carry in the futures);
3. annual change in federal government debt-to-GDP;
4. annual change in the federal government's budget deficit-to-GDP;
5. annual returns to spot gold;
6. annual returns to the S&P GSCI Commodities ex-Precious Metals Index;
7. annual returns to equities, represented by the S&P 500 Index;
8. annual headline U.S. CPI inflation rate.

All data, unless otherwise noted, are constructed on a monthly basis and available from March 1977 to October 2024 using data from Bloomberg, Federal Reserve Economic Data ("FRED"), Robert Shiller[6], and MeasuringWorth[7]. Where necessary, the data are lagged to account for the announcement delay, and are thus point-in-time. Next, we use equation 1 to

6. Online Data Robert Shiller. http://www.econ.yale.edu/~shiller/data.htm
7. Lawrence H. Officer and Samuel H. Williamson, "The Price of Gold, 1257 - Present." https://www.measuringworth.com/datasets/gold/

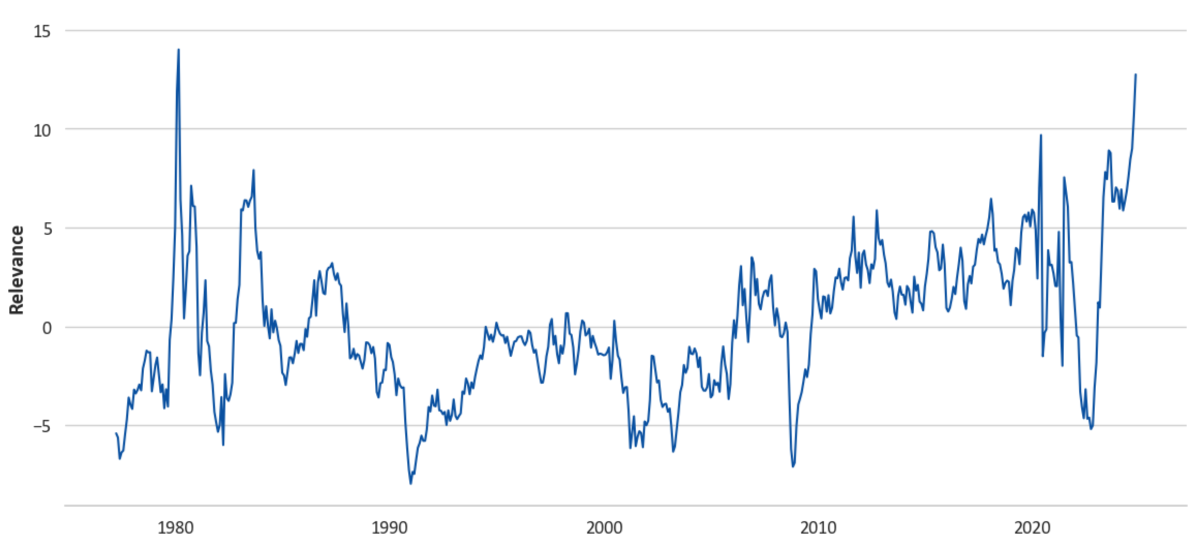
Exhibit 1: Sample of Relevance-Based Encoding Matrix							
Date	Mar. 1977	Apr. 1977	May. 1977	...	Aug. 2024	Sep. 2024	Oct. 2024
Mar. 1977	5.51	5.96	6.43	...	-3.34	-3.14	-5.41
Apr. 1977	5.96	6.48	7.01	...	-3.42	-3.13	-5.62
May 1977	6.43	7.01	7.76	...	-4.12	-3.90	-6.70
...	...	...	...	...	...	...	...
Aug. 2024	-3.34	-3.42	-4.12	...	6.72	7.79	8.99
Sep. 2024	-3.14	-3.13	-3.90	...	7.79	9.71	10.66
Oct. 2024	-5.41	-5.62	-6.70	...	8.99	10.66	12.74

Source: LongTail Alpha, Bloomberg, FRED, Robert Shiller, MeasuringWorth. October 31, 2024

calculate each time period's relevance to all other dates and construct the T x T encoding matrix; a portion of our encoding matrix is shown in Exhibit 1. Note that the RBEM is specific to the variables selected, but not gold itself; thus, no endogeneity issues exist by including gold as a variable itself.

Recall that higher positive values indicate greater relevance of the column date to the row date (and vice-versa, as the measure is symmetric), while lower negative values suggest little relevance. Two additional observations are in order. Firstly, the sample portion of the relevance-based encoding matrix presented in Exhibit 1 suggests that, perhaps unsurprisingly, dates closer to each current observation are more relevant than those farther away, for the primary reason that our variables are relatively slow-moving/overlapping. Secondly, a series of relevances to a particular data point, such as today (October 2024), naturally suggests regimes latent in the variables selected, even before applying k-means clustering to discover them. Exhibit 2 plots the relevance of each time period to today, showing that data in the early 1980s and immediately after 2010 are most relevant to the current observation, while data from parts of the 1990s and 2000s are not particularly relevant.

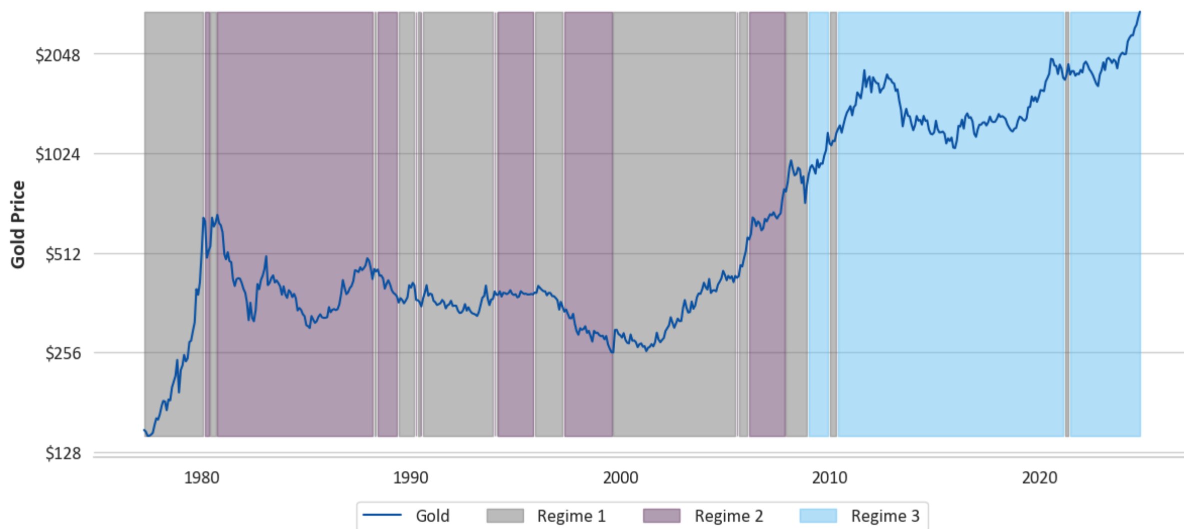
Exhibit 2: Relevance of Historical Periods to October 2024



Source: LongTail Alpha, Bloomberg, FRED, Robert Shiller, MeasuringWorth. October 31, 2024

We finally present the regimes identified from the RBEM of Exhibit 1 using first a dimensionality reduction exercise with PCA and then k-means clustering[8]. We impose the condition of three regimes, representing a common sense balance between parsimony and complexity. An alternative approach is to use a scoring metric to identify the optimal k that fits the data best, for example using the silhouette[9], Calinski and Harabasz[10], and Davies-Bouldin scores[11], or some combination thereof. The three regimes we identify using this procedure are shown in Exhibit 3. Note again that although we are applying the regimes to account for gold's performance and characteristics, they are not specific to gold itself but rather the totality of the variables selected in Mahalanobis distance space.

Exhibit 3: Gold Regimes Identified by k-Means Clustering on Relevance-Based Encoding Matrix



Regimes of gold. Regime 1 corresponds to gold as a "Real asset" regime; Regime 2 corresponds to gold as a "Commodity"; Regime 3 corresponds to gold as a "Stable Currency". Data sources: LongTail Alpha, Bloomberg, FRED, Robert Shiller, MeasuringWorth. October 31, 2024

Three aspects of the identified regimes immediately stand out. Firstly, they appear to be stable over secularly long periods of time, i.e. over periods as long as a decade. Secondly, they are mostly continuous and persistent, with monthly Markov transition probabilities of remaining in the same regime ranging from 93.8% for regime 1 to 98.9% for regime 3. Finally, the third regime (gold as "stable currency" regime) is very distinct from the other two, in that it only occurs for the first time in December 2008 after the start of the Global Financial Crisis ("GFC") and Great Recession, when the period of low interest rates began, pausing only briefly to revert back to the first regime in late 2009/early 2010 and from

8. We find similar results simply applying k-means clustering directly to the relevance-based encoding matrix.
9. Please see Rousseeuw 1987 for more details.
10. Please see Caliński and Harabasz 1974 for more details.
11. Please see Davies and Bouldin 1979 for more details.

March to May 2021, during the peak of fiscal expansion following the COVID-19 pandemic. Since May 2021, the variables used in the analysis imply that gold has been in the third, "stable currency" regime, possibly capturing the debasement of fiat currencies that followed exceptional monetary and fiscal stimulus and money creation.

As discussed previously, our intention with this new approach was to use the intuitive concept of relevance to define the regimes directly. We can measure whether we have succeeded in this goal by using the calculations of Kinlaw et al. 2023. They note that the statistical likelihood (according to a normal distribution) of regime r can be derived from equation 4.

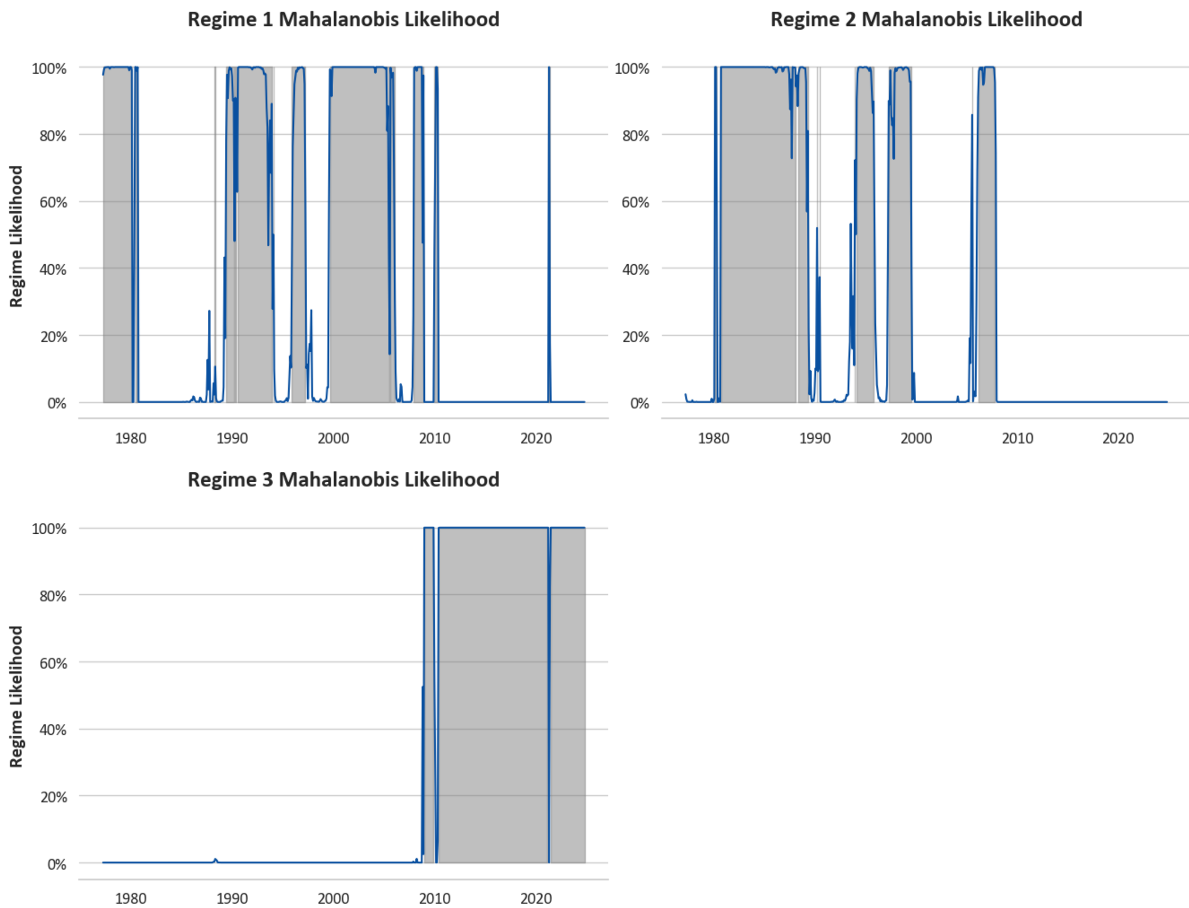
$$\xi_r(d_r) = (\det(2\pi\Sigma_r))^{-1/2}e^{-d_r/2} \tag{4}$$

In equation 4, *det* is the matrix determinant and e is the base of the natural logarithm, while d_r is the Mahalanobis distance defined by equation 3, with the exception that $\bar{x}$ is replaced by $\bar{x}_r$ and Σ is replaced by Σ_r, the average and covariance, respectively, of the independent variables within regime r. Following Kinlaw et al. 2023, we calculate the likelihood of each regime for all data points and then rescale these by the sum of all regime likelihoods, giving us a probability of each regime, p_r, given by equation 5.

$$p_r = \frac{\xi_r}{\Sigma_{all\ regimes\ i}\ \xi_i} \tag{5}$$

Based on equations 4 and 5, we calculate the probability of each regime and display these in Exhibit 4. The fit of the Mahalanobis likelihood of each regime to the regimes themselves is very tight. Although circular, since we are using the same variables employed in the regime identification to make a statement about how likely that regime is at a particular point in time, the result emphasizes that the regimes are indeed primarily based on the intuitive concept of relevance. Compared with Exhibit 8 in Kinlaw et al. 2023, our Mahalanobis likelihood fits are much tighter and nearly binary in nature, indicating that we have successfully used relevance to define regimes directly, the intended goal of our novel approach.

Exhibit 4: Mahalanobis Likelihood of Each Regime



Source: LongTail Alpha, Bloomberg, FRED, Robert Shiller, MeasuringWorth. October 31, 2024

We can also determine the importance of each underlying variable to a given point in time on a scale of 0-100% by taking the derivative of the regime's probability measure to changes in the $\boldsymbol{x}$ variables, as shown by Kinlaw et al. 2023. Equation 6 shows this derivative for regime r, which represents the sensitivity (in vector form) of p_r to each variable in $\boldsymbol{x}$. The total sensitivity for a selected data point is then the average of the three regime sensitivity vectors, each element of which we standardize by multiplying by the variable's standard deviation over the full sample. The normalized sensitivity vector is then rescaled by the sum of its components to yield a relative importance for each variable.

$$\frac{\partial p_r}{\partial x} = |p_r((\Sigma_{regimes\ i \neq r} p_i \frac{\partial d_i}{\partial x}) - (1 - p_r)\frac{\partial d_r}{\partial x})| \tag{6}$$

This analysis, along with examining the characteristics of each variable and that of gold during each state, reveals that the three regimes we have identified correspond to gold behaving as, respectively: 1) a real asset; 2) a commodity; and 3) a stable currency. The real asset regime is characterized as being primarily influenced by inflation, hence its name. The importance of this variable during the first regime is on average highest relative to inflation's

Exhibit 5: Average of Select Variables by Regime								
Regime	Gold Returns	Real Gold Valuation	Debt/GDP Change	Budget/GDP Change	Equity Returns	Real Rate	Real Rate Beta	Inflation
Real Asset	14.5%	2.8x	1.3%	-0.4%	6.5%	-0.1%	-0.01	4.6%
Commodity	3.4%	3.4x	1.1%	0.2%	14.8%	2.8%	0.08	4.5%
Stable Currency	6.9%	5.9x	3.1%	0.0%	9.4%	-1.3%	-0.30	2.4%

Source: Bloomberg, FRED, Robert Shiller, MeasuringWorth. October 31, 2024

importance during the other regimes, at nearly a quarter of the total sensitivity. Gold returns are highest in this regime, at 14.5% per year on average, a welcome level of diversification from traditional equity and bond investments during a period in which average yearly returns and nominal yields were only 6.5% and 4.5%, respectively, all while inflation averaged 4.6%, the most compared with the other regimes. The commodities regime is slightly more subtle, but is so named primarily because of: 1) the negative real compounded return that gold experienced on an annualized basis during this period, typical of commodities in contango based on the findings of Levine et al. 2018; 2) its high monthly correlation in both absolute and relative to other regimes terms to commodities of nearly 65%, compared with less than 20% in absolute value for the other regimes; and 3) its unusually positive (albeit small) average beta to both equities and real rates (here defined as the 3-month Treasury bill yield minus trailing YoY U.S. CPI inflation, in order to extend the data back farther), both of which indicate a more growth-oriented asset in line with commodities' long-run exposures. During this regime, average real rates were by far the highest relative to the other regimes, implying that gold's hedge against falling real rates perhaps only starts to kick in when they are closer to zero (either because of higher inflation or unsustainable government borrowing). The stable currency regime, which has only prevailed after the Great Recession, is characterized by the lowest volatility and most normal distribution of gold returns across any regime, as well as the richest average valuation of gold in real terms. Indeed, the volatility of its annual returns during this third regime is 15.5%, almost less than half that during the other two regimes, while gold's beta to real rates in the U.S. is most negative in this regime, suggesting protection against especially loose monetary policy. Exhibit 5 presents select variable averages across regimes, while Exhibit 6 depicts the monthly distributions of annual gold returns by regime, highlighting the regime summaries. Of note, various tests that examine whether two sets of data are drawn from the same empirical distribution, for example the two-sample Cramér–von Mises[12], Kolmogorov-Smirnov[13], and Anderson-Darling[14] tests, all indicate that the distributions of gold returns are statistically different from one another across regimes, implying that our regime methodology has successfully spread the distribution of gold's returns.

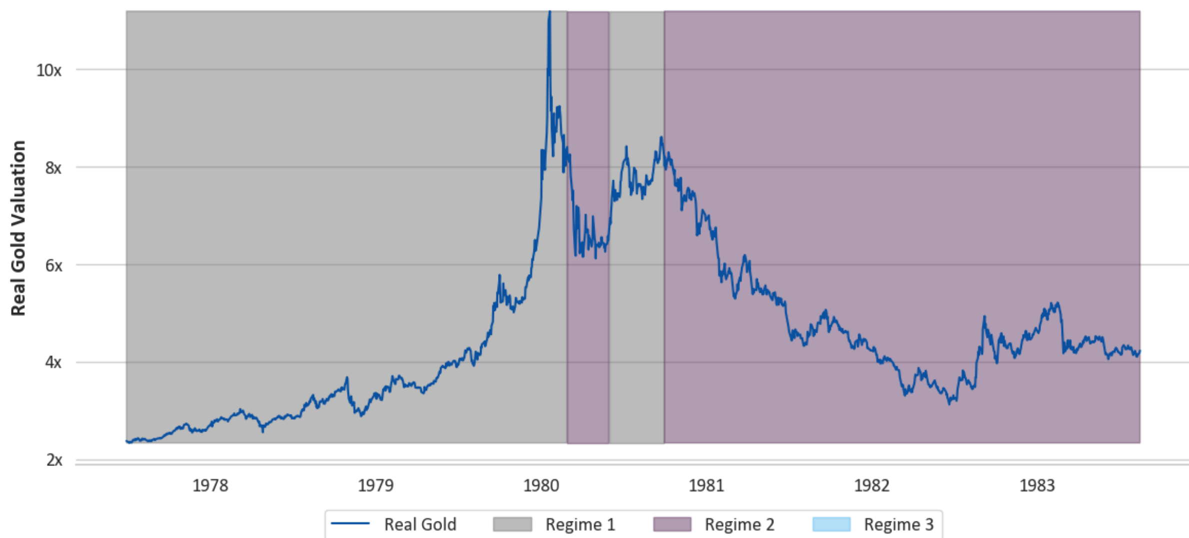
12. Please see Anderson 1962 for more details.
13. Please see Massey 1951 for more details.
14. Please see Anderson and Darling 1954 for more details.

Exhibit 6: Distribution of Annual Gold Returns by Regime

Source: Bloomberg, FRED, Robert Shiller, MeasuringWorth. October 31, 2024

Viewed in these regime contexts, we can now parse historical episodes to decompose and explain gold's performance. Take the example of the end of the second quarter of 1977 to August 16th, 1983, a nearly six-year period during which U.S. real rates rose dramatically (albeit not monotonically) from -2.3% to an all-time high of 7.2% (again when defined as nominal rates minus trailing one-year U.S. CPI inflation). Given the more recent experience of 2022, during which real rates similarly rose quickly by over 4%, causing gold to fall in real value by -4.9%, it might be assumed that gold's return would have been significantly negative. However, nominal gold prices actually rose 20.1% on an annualized basis compared with an 8.9% inflation rate. One of the primary drivers of this return was a valuation return of 10.3% annualized, from a starting ratio of gold price/CPI of 2.4x to an ending value of 4.2x, indicating a richening of the asset. While important, this full-sample view of valuation obscures the underlying dynamics throughout the period. In particular, up until September 1980 during the primarily real asset regime, real gold valuation rose to 8.2x, above its 99th percentile of all time, briefly touching 11.2x in January 1980 (well above the long-run median of 3.5x). Inflation during this period was 10.9% annualized and reached 15.7% YoY at its peak, and gold, true to its behavior as a real asset, delivered a 63.6% annualized return, driven in large part by a 47.5% valuation return as investors sought to protect the real value of their portfolios. Indeed, inflation was very important to determining this regime in Mahalanobis likelihood space based on the variable importance of Kinlaw et al. 2023, with an average importance of over 28%, higher than at nearly any other time in history. Subsequently, after September 1980, the commodities regime prevailed through to the end of the sub-sample. Along with inflation and other commodities, gold retreated back to a more modest valuation level, with inflation falling to a mere 2.5% YoY by the end of the period as the Volcker Fed held nominal rates high to slow economy-wide price increases. The identified regimes, solely determined by relevance, were able to account nicely for gold's performance during a very volatile macroeconomic and inflationary period. Exhibit 7 plots the real price of gold against the backdrop of the regimes we have identified during this period.

Exhibit 7: Real Gold Valuation v. Identified Regimes from Q2 1977 to August 16th, 1983



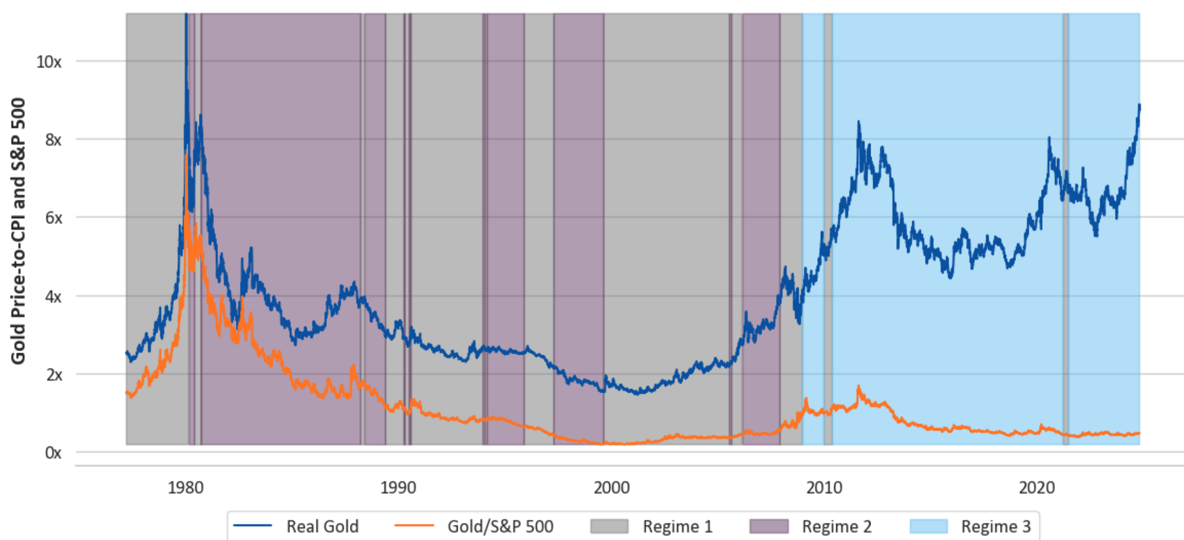
Source: Bloomberg, FRED, Robert Shiller, MeasuringWorth. October 31, 2024

Another important era in gold's history is from August 25th, 1999, to August 22nd, 2011, a long 12-year period during which gold went through one of its strongest bull markets ever (with the exception of the inflationary 1970s, when gold generated its highest returns by decade of about 30% annualized). During this period, gold delivered annualized returns of 18.3%, composed of a significant increase in valuation from approximately 1.5x to 8.4x which contributed 15.4% toward that return. Importantly, such strong returns came during an era when equities generated a negative price return and only a slightly positive total return, as the period spans from the height of the Dot-Com Bubble to the post-Great Recession recovery. During this period the predominant regime was the real assets regime, which prevailed for nearly two-thirds of the time. In the commodities regime during this period, which occurred between 2005 and 2007 immediately prior to the GFC, gold was highly correlated to a broad basket of commodities, as mentioned previously. Despite this, and the fact that commodities fell over -72% during the Great Recession[15], gold's drawdown was a much more modest -29.0%, and over the whole recession it actually gained 16.6% cumulatively on the back of a 14.5% valuation return as investors fled to safety, with both commodities and equities down over a third. Part of the reason for this is that the stable currency regime began in December 2008 and saw an acceleration of gold's returns as its valuation increased from 4.1x to 8.4x amid a backdrop of the FOMC's first two rounds of quantitative easing along with Operation

15. National Bureau of Economic Research: "US Business Cycle Expansions and Contractions". https://www.nber.org/research/data/us-business-cycle-expansions-and-contractions

Twist[16]. Two measures of gold's valuation over the long run are shown in Exhibit 8; the first is our preferred measure of real gold, while the second is gold price/S&P 500, which highlights gold's current relative cheapness to equities compared with the late 1970s/early 1980s period. In the exhibit, the substantial increase in valuation during the 2000s is clearly visible as the transition to the stable currency regime occurred, with investors bidding up the value of gold amid unprecedentedly low global interest rates and a coordinated debasement of fiat currencies. As of October 2024, gold is the most richly valued on a real basis (at the 99.8th percentile) it has been in this nearly 50-year history, with the exception of early 1980.

Exhibit 8: Gold's Valuation Relative to U.S. CPI and the S&P 500 Index



Source: Bloomberg, FRED, Robert Shiller, MeasuringWorth. October 31, 2024

These two example episodes illustrate the usefulness and power of our new regime identification methodology. Note that several of the variables we have explored above were not explicitly included in the model; for example, the model had no direct knowledge of real rates or gold's beta to real rates and other asset classes like commodities, and yet found that the most likely regimes - in a Mahalanobis likelihood sense, as described in equation 5 - corresponded to very different types of behavior along these dimensions.

Implications for Asset Allocation

Of course, while the prior results are no doubt interesting and illustrative of our new technique, the natural next question to ask is whether information in our regimes can be used to make strategic and tactical asset allocation decisions. One simple yet highly generalizable

16. Federal Reserve Bank of New York: "Large-Scale Asset Purchases". https://www.newyorkfed.org/markets/programs-archive/large-scale-asset-purchases

Exhibit 9: Suggested Allocation to Stocks, Bonds, and Gold by Regime in MVOs as of October 2024					
Regime	Regime Probability	Equity Allocation	Bond Allocation	Gold Allocation	Expected Sharpe Ratio
Real Asset	1.1%	30.9%	54.5%	14.6%	0.67
Commodity	0.0%	59.3%	39.5%	1.2%	0.92
Stable Currency	98.9%	59.5%	33.8%	6.7%	0.41
Weighted Average		59.2%	34.0%	6.8%	0.41

Source: Bloomberg, FRED, Robert Shiller, MeasuringWorth, SBBI from the CFA Institute. October 31, 2024

way to quantify this is to: 1) identify the mean-variance optimal ("MVO") portfolio in each regime based on the historical performance of a select set of asset classes within each period; and 2) take the weighted-average MVO portfolio across regimes using the current set of Mahalanobis likelihood probabilities, adjusted for the long-run Markov transition probabilities. We make use of this procedure in this section.

For simplicity, we consider a three-asset portfolio composed of U.S. equities, long duration U.S. Treasuries, and gold. Data for the former two comes from Ibbotson's Stocks, Bonds, Bills, and Inflation ("SBBI") dataset[17]. We calculate the historical average returns, volatilities, and correlations between these three assets during each regime and use these as our inputs for a mean-variance optimization exercise[18]. Due to issues with MVO caused primarily by errors in means raised by Chopra and Ziemba 1993, we employ a resampling methodology in which we perturb the expected returns in each state proportional to their expected volatilities and optimize 1,000 times, taking the average of the results to arrive at the final recommended allocation between the three asset classes in each state. The suggested allocation to each asset class based on this resampled MVO approach is given in Exhibit 9 for not only each regime, but also on a weighted average basis based on the current probabilities. That overall recommended asset allocation matches the stable currency regime closely, as the current probability of that regime prevailing is almost certain for the next few years.

Although the stable currency regime is currently considered to be most likely, Exhibit 9 highlights two things. Firstly, the suggest allocations in the commodity and stable currency regimes are quite similar to one another. The optimal commodity regime portfolio resembles something very close to a traditional 60-40 asset allocation, while the stable currency MVO portfolio adds an allocation to gold because of the flight-to-safety property the asset has brought in a period of generally loose fiscal and monetary policy. Indeed, the importance of the two fiscal policy variables, namely changes in the federal government's debt-to-GDP ratio and its budget (deficit)-to-GDP ratio, measured using equation 6 suggests that policy contributed materially to the determination of the stable currency regime; at a joint importance level of nearly 15%, these two variables together are more important than all other variables in our model, except real gold valuation - reflecting demand for the asset in a low rate environment - and inflation - which was low and stable throughout most of the 2010s, itself a major driver of low rates - which together yield information about nominal interest rates influenced by monetary policy. Secondly, should the stable currency regime shift to the real asset regime, the optimal portfolio could change dramatically. In the real asset regime, the optimal portfolio shifts materially toward gold and bonds and away from equities.

17. CFA Institute. "Stocks, Bonds, Bills, and Inflation (SBBI) Data" https://rpc.cfainstitute.org/en/research-foundation/sbbi

18. We force the allocation to each asset class to be between 0% and 100% (i.e., no shorting or leverage) and aim to maximize Sharpe ratio in the MVO resamples.

Conclusion

Identifying regimes for the purposes of ex-post attribution or ex-ante forecasting is an important component of understanding the drivers of returns across financial markets. We develop a new methodology for regime identification that attempts to maintain the advantages of more complex models of regimes, namely capturing complexity and non-linearity, while simultaneously doing so with an intuitive and transparent approach. On a spectrum of more parsimonious to more sophisticated models, ours falls squarely in the middle. It is based on the new concept of relevance, itself an active area of research, which states that, when it comes to the task of prediction, the more similar an historical observation and the more unlike its long-term average, the more useful that data point is. Couched in this setting, our regimes offer a way to identify the most statistically relevant periods to one another in a highly generalized and robust way.

We apply our new methodology to one of the world's oldest diversifying assets, gold. In doing so we identify three "faces" of the asset, in that it can behave as either: 1) a real asset; 2) a commodity; and 3) a stable currency. Emphasizing the general nature of our methodology, several characteristics of each regime were not directly known to the model, yet were still captured by the regimes it identified. For example, the fact that gold's beta to real rates in the second regime was positive, a highly unusual state of affairs for gold, was found to be a key driver of the commodity regime.

Our approach offers a simple, yet also highly generalizable way to proceed with the task of strategic and tactical asset allocation in the presence of regimes. Taking the example of a simple equity and fixed income portfolio like the traditional 60-40 approach, we find that should the current stable currency regime shift to the real asset regime, the implications for strategic asset allocation would be significant, with an increased allocation to gold and a decreased allocation to equities. This finding highlights the importance of not only understanding the historical performance of different asset classes by regime, but also of preparing for possible future regime shifts which usually bring heightened volatility across asset classes.

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