Option Total Return and Active Option Portfolio Management

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Abstract

We introduce the concept of total return for options and option portfolios. By taking local derivatives of the option price with respect to the underlying drivers in an option pricing model (i.e. delta, gamma, vega, theta), we can decompose daily option price changes into the sum of the changes in these underlying drivers. By accumulating the contribution from the underlying drivers, we can compute each driver's contribution to the total performance of a portfolio of options. We discuss the relevance of this framework to applications such as tail risk hedging, and discuss how the approach can be useful to the active management of options portfolios. Our approach provides a natural extension of the concept of total return that is widely used for evaluation of traditional investment portfolios.

Highlights

- We introduce the concept of total return for options and option portfolios. By keeping track of the local contributions from the drivers of the options price and accumulating these contributions, we can attribute the total return to changes in these key drivers for a given horizon.
- By attributing the total return of various option structures in terms of the underlying drivers, we highlight how our approach provides important details on the trade offs involved in choosing different option structures to attain a given objective.
- With an example from the the COVID-19 crisis, we demonstrate the usefulness of this framework to active option portfolio management such as swapping from vanilla options to spreads.

The concept of "Total Return" is critical for active investment management. In the simplest terms, total return means that the returns of an investment should be evaluated by computing returns from both static and dynamic drivers of price changes, i.e. the return from combining both the passage of time and from the change in the underlying drivers of the price. In the bond markets, the static returns originate from the yield of the bond, and the dynamic returns originate primarily from changes in yield curve level, shape and curvature (see for example Bhansali 2011 for how this is used in practice in bond portfolios). If we think of the price sensitivity of a bond as emanating from the partial derivatives of the bond's price with respect to the factors that drive the price, then we can also decompose the total return of the bond into the return attributable to these derivatives. For instance the modified duration of a bond represents the percentage change of the bond's price due to a parallel shift in the yield curve. So the total return contribution of the duration "factor" can be used to explain how much of the return arrived from such parallel shifts. Other derivatives-based price sensitivities can similarly be translated into their contributions to the total return.

We can also interpret these results in a different way. When applied to fixed income portfolios, the return of a bond holding can be decomposed into two terms - return from the things that don't change (i.e. yield), and return from the things that do change (i.e. change in level, shape, curvature, spread, convexity etc.) Indeed, this approach has become a foundation of fixed income portfolio management, and has resulted in enormous success for firms like PIMCO whose founder Bill Gross was one of the first investors to apply these ideas to the management of bond portfolios, and which at one point resulted in the PIMCO Total Return Fund becoming one of the largest actively managed mutual funds. Once bonds were understood to be dynamic objects rather than static buy and hold investments, active management of bond portfolios literally took off, since the return from active management of bond portfolios could potentially be higher or lower than the yield of the portfolio one would realize simply by holding the underlying bonds to maturity. With trillions of bonds outstanding today, and thousands of active bond managers, the current state of affairs is one where managers try to manage their portfolios to deliver "alpha" against bond market benchmarks which are basically passive buy and hold investments. The key to this approach is to be able to attribute returns to the underlying drivers of return. For instance, if parallel shifts of the yield curve are responsible for generating excess returns, and once believes the level of the yield curve to be mean-reverting, e.g. because of its relationship to inflation, then the decomposition would allow one to focus on this underlying driver of returns.

The total return concept also had major consequences for risk mitigation and diversification. By using risk factors such as duration and "curve" duration, and "spread" duration, bonds have evolved from being passively held diversifiers against risky markets such as equities to essential return generating ingredients of portfolios. With the advent of derivatives such as futures, swaps and options, and the ability of leverage that these instruments provide, bonds are now also used in systematic approaches, such as risk-parity constructs, to provide more balanced and robust portfolios.

The concept of total return thus allows investors to have a more complete toolkit with which to evaluate the trade-offs of holding a given combination of positions. Indeed, it is possible for a bond position to lose money in price terms, i.e. have negative price returns, but as long as the yield and carry are sufficiently positive, the same position can lower the negative total return, perhaps delivering even positive total return. The total return approach thus provides a more refined approach to understanding bond performance.

Note that the concept of total return is not limited to bonds. Even equity market returns can be understood by decomposing the returns of a stock into static and dynamic components, such as dividend income and the return from stock price changes respectively. For currencies, the total return concept can be applied similarly by decomposing into return from changes in the spot exchange rate, and return from the interest rate differentials of the currency and the counter-currency. For commodities, the total return is simply the sum of the price return of the commodity added to the roll-down or roll-up that results from commodity curves in contango or backwardation.

Thus we see that the concept of total return is quite universal to core assets and asset classes. The application to option portfolios is straightforward, as long as we can identify the drivers that drive option prices and compute local derivatives of price with respect to those drivers. In the Black-Scholes equation, these partial derivatives are commonly known as "Greeks". The concept of return from things that change and from things that don't change in the context of options is also straightforward since options have a natural concept of time decay ("Theta"). To our knowledge, this total return decomposition approach has not been applied previously in a consistent manner to options portfolios. Part of the reason is that options are more complex to price and higher frequency (e.g. daily) data on all the strikes and expiries, with their corresponding "greeks" has not been available until recently. Another, perhaps more important reason, is that the concept of active management of options has only recently started to come to a state of maturity. As was the case prior to the innovations that lead to bonds being evaluated from a total return perspective, until recently options have been thought as instruments to be held to maturity, with little attention paid to the return that can be generated by options at intermediate points prior to expiration. However, with providers such as Option Metrics and Bloomberg offering readily available option price and sensitivity data, researchers are now able to use the techniques of total return calculations for options portfolios.

The option total return approach immediately yields three benefits similar to the benefits for bond portfolios discussed above. First, the total return concept for options allows us to quantify the tradeoffs from the different greeks for a given horizon; second, the strategy allows us to include options in multi-asset portfolios to better manage risks of such portfolios, especially since options inherently provide explicit, asymmetric leverage. Finally, the total return concept allows one to potentially create relative value trades to deliver alpha, depending on exposure to which underlying greek is preferred.

In this paper we first define the concept of total return to options and option portfolios, and then illustrate in the context of a Black-Scholes environment how these techniques can be used to manage option portfolios. For a long option position, the total return is the sum of the return from the movement of the underlying factors that drive the prices, e.g. primarily the change in the underlying asset, the change in volatility, and the passage of time, i.e. time decay. Normally the change in option prices are understood by computing the local derivatives of the Black-Scholes price with respect to the fundamental variables: price, time, and implied volatility; these derivatives are commonly known by names such as delta, gamma, vega, theta etc. Options are, however, quite non-linear and these local derivatives ("Greeks") change with time. This means that for a proper analysis, one has to compute local derivatives at a high frequency, and then utilize a sum or time integral to compute cumulative contributions from the various greeks. We do exactly this in this paper. To show the power of this technique, we utilize our approach to attribute return to various greeks for tail hedging portfolios, and a long-short relative value portfolio of S&P 500 index options. We note that while the concepts are illustrated using the Black-Scholes formula, the approach can easily be extended to more complex option pricing models.

FRAMEWORK

We decompose the total return of an option into the return attributable to each of the greeks: delta, gamma, vega and theta. We ignore rate sensitivity ("rho") and higher order Greeks, though this approach can easily be extended to incorporate them. Delta, in the Black-Scholes equation, refers to the local derivative expressing the change in option price for a given change in the price of the underlying: $\frac{\partial P}{\partial S}$. Correspondingly, the option return attributable to Delta is this partial derivative multiplied by the change in the underlying: $\Delta P_s = \frac{\partial P}{\partial S} * \Delta S$. Generalizing to all greeks, the change in option value ΔP between time t and $t + \Delta t$ can be expressed as follows (we use the italicized full names to signify the contribution to the total price change of the option from the various individual components):

$$\Delta P_{t,t+\Delta t} \approx \Delta delta_{t,t+\Delta t} + \Delta gamma_{t,t+\Delta t} + \Delta vega_{t,t+\Delta t} + \Delta theta_{t,t+\Delta t} \tag{1}$$

where:

$$\Delta delta_{t,t+\Delta t} \equiv \delta_t * \Delta S_{t,t+\Delta t} \tag{2}$$

$$\Delta gamma_{t,t+\Delta t} \equiv \frac{1}{2} * \gamma_t * (\Delta S_{t,t+\Delta t})^2 \tag{3}$$

$$\Delta vega_{t,t+\Delta t} \equiv \nu_t * \Delta \sigma_{t,t+\Delta t} \tag{4}$$

$$\Delta theta_{t,t+\Delta t} \equiv \theta_t * \Delta t \tag{5}$$

S denotes the price of the underlying; $\delta, \gamma, \nu, \theta$ denote the Black-Scholes delta, gamma, vega and theta, respectively (see Black and Scholes 1973). The interval frequency Δt could be whatever we wish, but given that our historical option data from OptionMetrics is provided on a daily frequency, we let $\Delta t = 1$ day. Letting T denote the number of days until expiration, we can decompose the lifetime change in the value of an option by summing up all interim (daily) price changes:

$$\begin{aligned} \Delta P_{0,T} &= \sum_{i=1}^{T-1} \Delta P_{i,i+1} \\ &\approx \sum_{i=1}^{T-1} \Delta delta_{i,i+1} + \Delta gamma_{i,i+1} + \Delta vega_{i,i+1} + \Delta theta_{i,i+1} \end{aligned} \tag{6}$$

By summing up only the greek component of interest, we can use this equation to find each greek's contribution to an option's lifetime change in value. Note that we lmiit our derivatives to the first order, but the approach is easily extendible to higher order derivatives if needed.

EXAMPLES

To demonstrate our approach in various settings, we compute the contribution to the option total return from various 1-year option portfolios on the S&P 500, for a historical sampple period starting in 1996. All data is from OptionMetrics. Each strategy buys 1-year tenor options on a quarterly basis. Strategies 1-4 size each new option position such that the delta of the new position is equal to 0.025 at the portfolio level. Since options are bought quarterly, the strategies reach a stable state of a "ladder", with 4 separate options positions roughly 3/6/9/12 months away from expiration, and a combined delta of roughly 0.1 (0.025 *4) at the total portfolio level. Strategy 5 simply buys 1 contract of the put spread, and sells the requisite number of call options to match the premium spent.

In each case K refers to option strike price, and S denotes spot price. ATM and OTM refer to at the money, and out of the money respectively. OTM puts have $K < S$, and OTM calls have $K > S$. ATM means $K = S$ regardless of option type. Moneyness is defined as $\frac{K}{S}$.

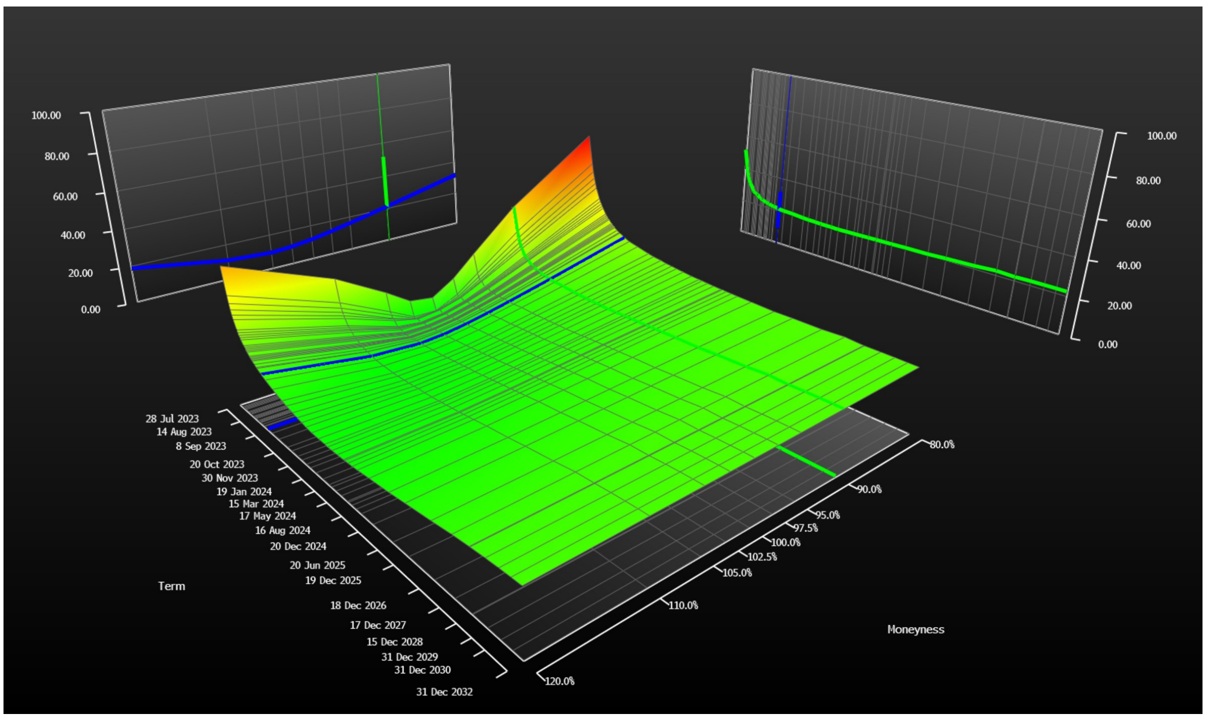
In each chart below, we rebase the starting value of the portfolio to $100, so that we can easily compare the total returns from each source of positive or negative return.

- **Strategy 1**: At the Money Puts: Purchases 1 year ATM puts (P100).
- **Strategy 2**: Out of the Money Puts: Purchases 1 year 20% OTM puts (P80).
- **Strategy 3** Out of the Money Calls: Purchases 1 year 10% OTM calls (C110).
- **Strategy 4**: Put-Spread Collar: Purchases one P90-P80 put spread by buying 10% OTM put and selling one 20% OTM put. The strategy then sells the requisite quantity of 10 % OTM calls (C110) to fund the purchase price of the put spread.
- **Strategy 5**: Tail Hedged Portfolio: Allocates the entire starting capital to the S&P 500. At the beginning of each quarter, a 20% OTM put is bought in a quantity such that the option's notional equals 25% of the value of the portfolio's S&P 500 position at that time (note that for illustration we assume that these options are bought on free margin, i.e. without interest expense, and the quantity of S&P 500 never changes). The cumulative cost of these options, subtracted from the value of the S&P 500 position, equals the value of the portfolio/strategy.

For the options held in a portfolio at any moment, we utilize the framework above to decompose each option's daily price change into contributions from delta, gamma, vega, and theta. The total realized return across all positions is denoted by "Strategy PnL". Our total return framework utilizes a time integral of first and second order local derivatives of the Black-Scholes equation, and while this will capture a significant amount of the total deviation in option price, it will not capture all of it. It may be possible to capture more deviation by substituting in a better options model for Black-Scholes, and using higher order terms. With this in mind, each portfolio's realized return in the backtest will not necessarily equal the explained return (the sum of the calculated greek contributions). The cumulative sum of each greek's individual contribution is named accordingly.

In exhibit 1 we show a sample volatility surface for the S&P 500 that summarizes the interaction of strikes, expiries and implied volatility. The shape of the surface is important in understanding how some of the total return contributions evolve as the underlying moves, and options get closer to expiry. In particular, readers should recall that generally puts trade at higher volatility than calls with the same out of the moneyness, and the rate at which the volatility increases is more rapid for shorter expiry options.

Exhibit 1: Implied Volatility Surface

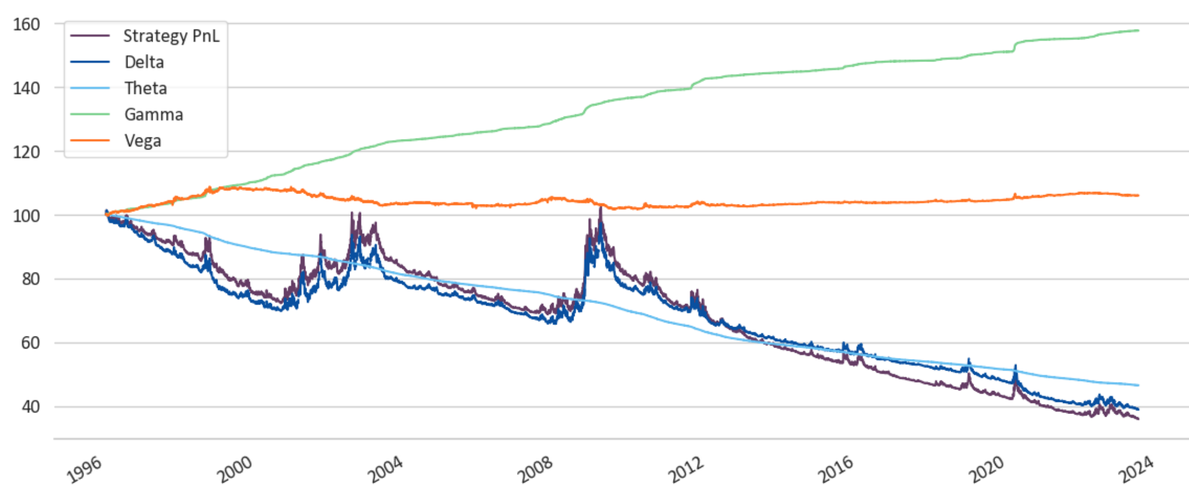


Source: Bloomberg. As of July 27, 2023

At the Money Puts (P100)

For a simple strategy containing at the money puts, we see in exhibit 2 that over the sample historical period considered, the net total return has been negative, since the S&P 500 has rallied over this period. Thus the delta (market) contribution has been negative, in addition to the theta (time-decay) contribution. The contribution from gamma is roughly the opposite of the contribution from theta; in trading circles this relationship is known as "theta is the price of renting gamma". Interestingly, the vega contribution is slightly positive. This can be traced back to the definition of the vega contribution. As the S&P 500 goes up, an at the money put becomes increasingly out of the money; since deeply out of the money puts tend to have a higher implied volatility than at the money puts (please refer to exhibit 1), and vega is always positive, the volatility return contribution is therefore positive. Further, and referring again to exhibit 1, as an out of the money put's expiry shortens, its implied volatility tends to rise further.

Exhibit 2: At The Money Put (P100)

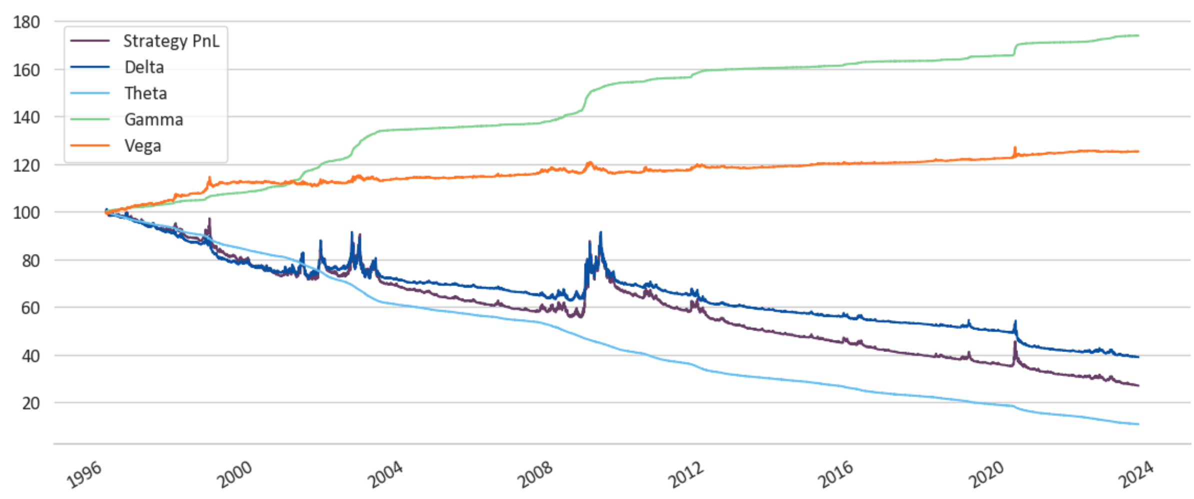


Sources: LongTail Alpha, OptionMetrics

Out of the Money Puts (P80)

For out of the money puts, we see in exhibit 3 that over the historical period considered, the total return has been negative since the S&P 500 has rallied. Thus the delta (market) as well as the theta (time-decay) contributions have been negative. Since out of the money puts have less initial delta than at the money puts, in order to target the same delta (0.025 at the portfolio level), this strategy needs to purchase more options. In contrast with the at the money put strategy, the negative contribution of theta is now more than the negative contribution of delta. Both the at the money put strategy and this strategy lost roughly the same amount from delta, but this strategy lost almost twice as much from theta. However, it gained more from gamma and vega showing that more out of the money options are more sensitive to jump risk (recall that gamma is the rate of change of delta, so it is proportional to jump risk). Again, the contribution from gamma is roughly the opposite of the contribution from theta. The vega contribution is positive. This can be traced back to the definition of the vega contribution, and put-call parity. As the underlying rises and out of the money puts become more out of the money, we know that out of the money calls become less out of the money, and possibly even in the money. Since put-call parity requires the implied volatility of an out of the money put to be the same as an in the money call, this results in the implied volatility of an out of the money put rising as the market rallies and the option expiry shortens.

Exhibit 3: 20% Out of the Money Put (P80)

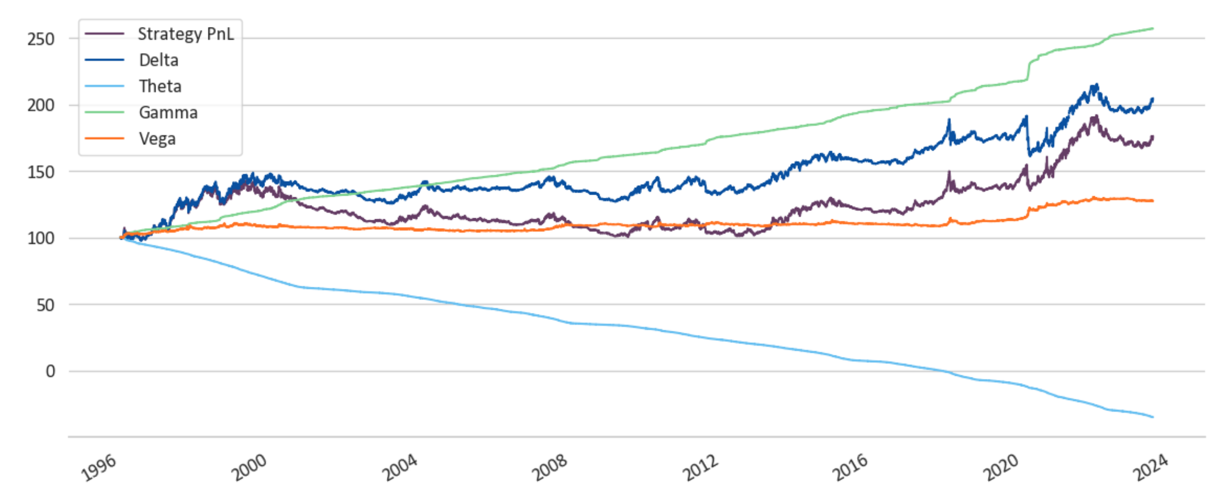


Sources: LongTail Alpha, OptionMetrics

Out of the Money Calls (C110)

For out of the money calls, we can see in exhibit 4 that over the historical period considered, the net total return has been positive since the S&P 500 has rallied. Thus the delta (market) has been positive while the theta (time-decay) contribution is still negative since as a long option, the calls still lose time-value. The contribution from gamma is roughly the opposite of the contribution from theta. Interestingly, on a relative basis, the vega contribution is more positive than the at the money puts example as well. This can be traced back to the definition of the vega contribution; deeply out of the money puts tend to have a higher volatility than at the money puts. Further, as the option's expiry shortens, the volatility of the out of the money puts tends to rise rapidly, which through put-call parity, results in increases in the volatility of in the money calls.

Exhibit 4: 10% Out of the Money Call (C110)

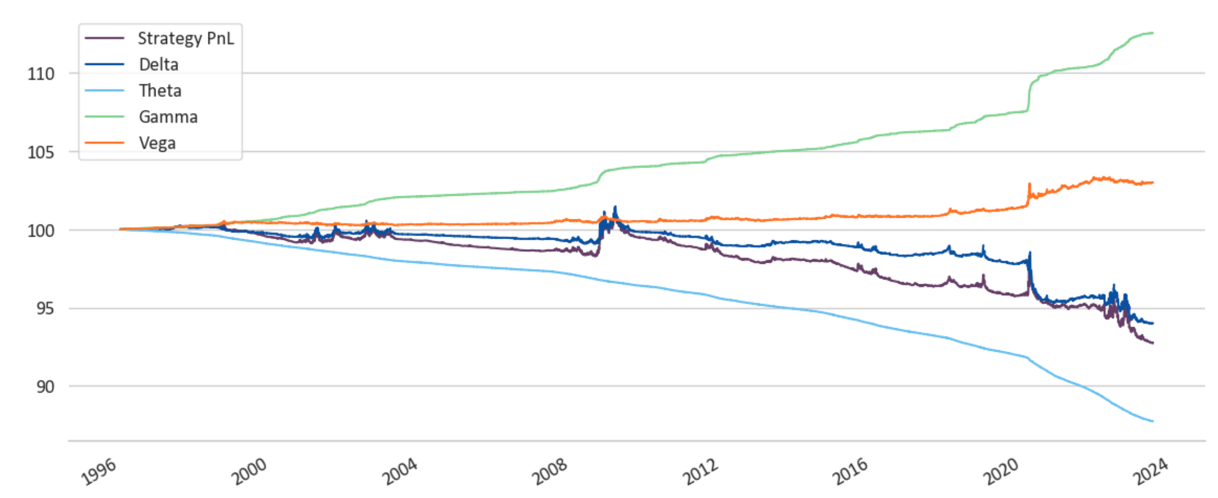


Sources: LongTail Alpha, OptionMetrics

Put-Spread Collar: P90-P80 funded with short C110

When we fund the purchase of a put-spread with the sale of a call option, we see that over our sample historical period the total return is negative, as shown in exhibit 5. This occurs because both the time decay and delta (market) contributions are negative.

Exhibit 5: P90-P80 funded with short C110

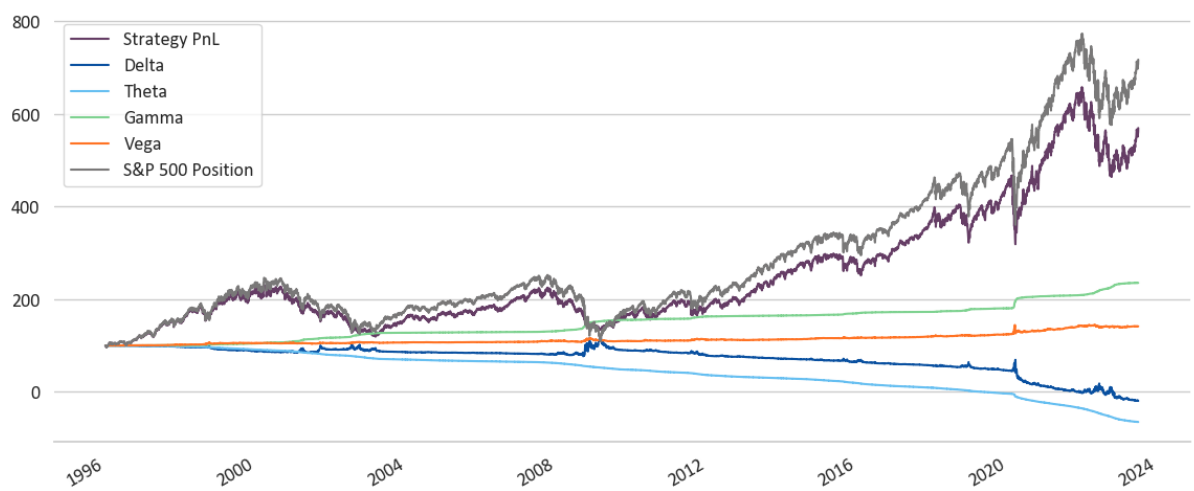


Sources: LongTail Alpha, OptionMetrics

Tail-Risk Hedge: S&P 500 hedged with 20% OTM Put (P80)

For a tail-risk hedged portfolio, we see in exhibit 6 that the contribution from time decay results in negative total return contribution to the hedged portfolio. The hedged portfolio's total return is lower than the total return of an unhedged S&P 500 position, but as a compensation, the tail hedge provides protection against severe drawdowns. This can be seen in exhibits 7 and 8 which show performance during the financial crisis (2007 - 2010) and the COVID-19 crisis (Feb - March 2020).

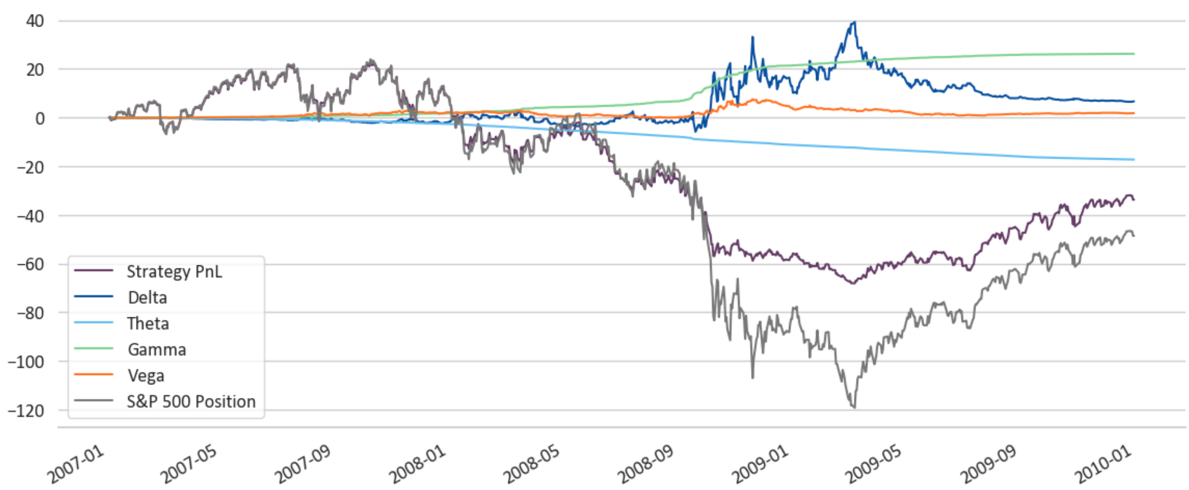
Exhibit 6: S&P 500 hedged with 20% OTM Put (P80)



Sources: LongTail Alpha, OptionMetrics. SPX Index rebased to 100

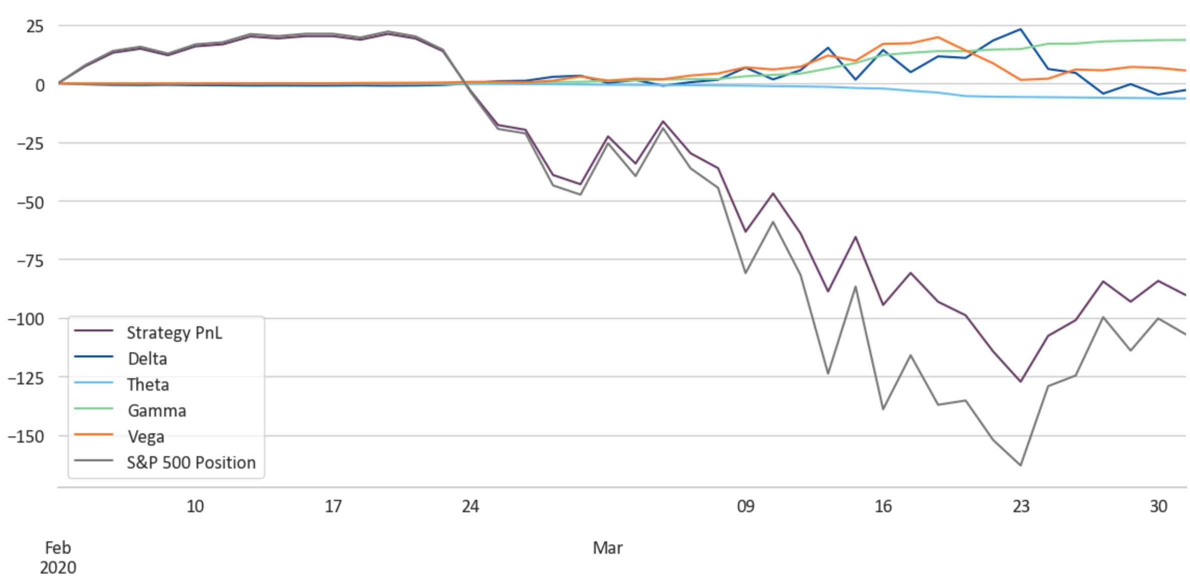
In exhibits 7 and 8 we highlight the contributions to the hedged portfolio performance during the Great Financial Crisis and the COVID-19 Crisis. As can be seen from these exhibits, once we zoom into specific crisis periods, the negative contribution to total return from the time decay of the hedges is more than made up for by the contribution to total return from the delta, gamma and vega effects.

Exhibit 7: Hedged - Great Financial Crisis



Sources: LongTail Alpha, OptionMetrics. Jan 3, 2007 - Jan 1, 2010

Exhibit 8: Hedged - Covid Crisis



Sources: LongTail Alpha, OptionMetrics. Feb 1, 2020 - April 1, 2020

The table shown in exhibit 9 depicts the decomposed performance of the example strategies. Note that for easy comparison, all total returns have been normalized to the same units (return in dollars relative to a starting value of $100).

Exhibit 9: Strategy Total Return					
	P100	P80	C110	S&P 500 Hedged (P80)	P90-P80 w/ C110
Delta	-64.38	-66.55	85.13	-129.42	-71.17
Theta	-56.75	-94.65	-154.54	-174.93	-133.83
Gamma	54.33	68.35	137.50	125.26	114.06
Vega	2.69	19.80	8.06	31.44	18.59
S&P 500	-	-	-	616.96	-
Total	-64.12	-73.05	76.16	469.32	-72.35

Jan 1, 1996 - Jun 30, 2023

In the S&P 500 tail-hedged strategy, we separate the return attributable to the S&P 500 position and the options positions during two periods of interest: the Great Financial Crisis and the COVID-19 Crisis. Results for the Great Financial Crisis are shown in exhibit 10.

Exhibit 10: Great Financial Crisis			
	S&P 500 Position	Options Positions	Combined Portfolio
Delta	-48.57	6.00	-42.57
Theta	0.00	-17.87	-17.87
Gamma	0.00	25.55	25.55
Vega	0.00	1.16	1.16
Total	-48.57	14.84	-33.73

Jan 3, 2007 - Jan 1, 2010

During the COVID-19 Crisis, shown in exhibit 11, we see that the options positions on net lost money due to delta. As the S&P 500 fell, the delta on the put options became increasingly negative. Combined negative delta across all options peaked on March 23rd, and March 24-26 saw three days of explosive gains in the S&P 500: 9.4%, 1.1%, and 6.2%. In other words, just as the options were most sensitive to moves in S&P 500, the index moved sharply to the upside as the Fed started to buy assets through a new round of quantitative easing, and also cut interest rates sharply. However, these large magnitude moves in the S&P 500 led to large option gains from gamma exposure, and the resulting upward shift in implied volatility lead to gains from vega exposure. These results thus show us a “X-ray” of which components of the options portfolios resulted in gains during these episodes and which ones resulted in losses, and in our view provide a refined attribution of which features of options tend be more potent in periods such as these.

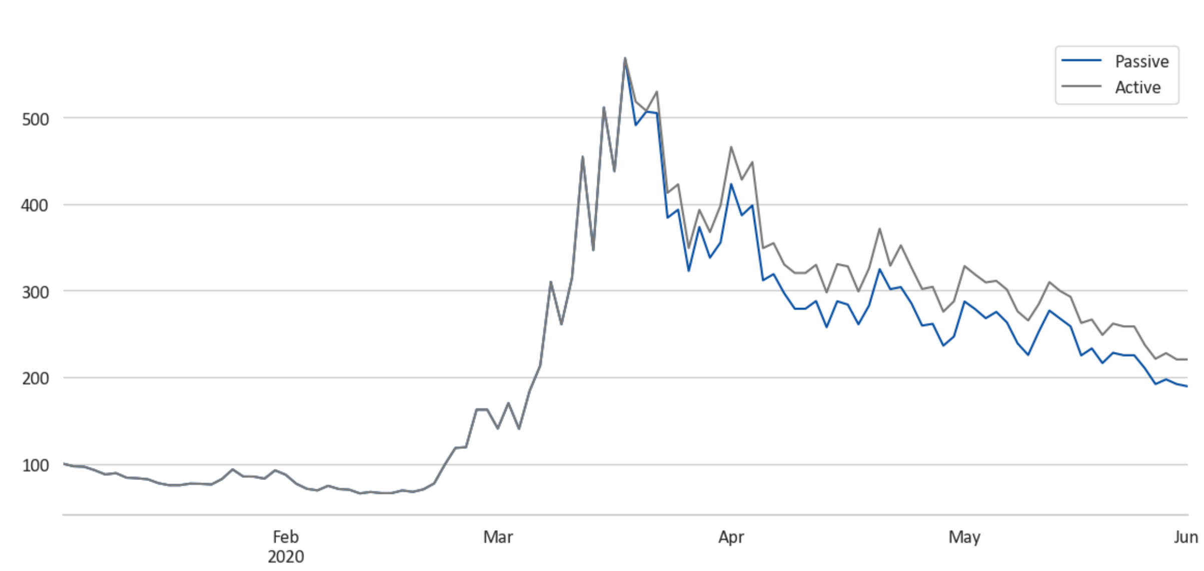
Exhibit 11: Covid Crisis			
	S&P 500 Position	Options Positions	Combined Portfolio
Delta	-107.02	-2.32	-109.34
Theta	0.00	-5.95	-5.95
Gamma	0.00	19.01	19.01
Vega	0.00	6.00	6.00
Total	-107.02	16.75	-90.27

Feb 1, 2020 - April 1, 2020

ACTIVE MANAGEMENT

In this section we demonstrate the benefits of actively managing option portfolios using the decomposition we introduced previously. We compare two strategies, one passive and one active, during the covid crisis of 2020, as shown in exhibit 12. Both strategies purchase a single 20% OTM S&P 500 put on January 3rd, 2020. The passive strategy simply holds the option to expiry. The active strategy mimics an active option strategy with a 5x monetization threshold; accordingly, this strategy monetizes the put at a 5x multiple, which occurs on March 18th, 2020 and redeploys this entire premium into a put spread with the same delta and premium[2]. Below we plot the performance of these two strategies.

Exhibit 12: Active Monetization into Put Spread



Sources: LongTail Alpha, OptionMetrics. Rebased to 100

We find it insightful to detail the performance not only over the full sample period, during which the S&P 500 decreased 6.2%, but also before and after the active strategy's monetization in mid-March, when the S&P 500 first decreased 26.4%, and then increased 27.4%, all within 6 months. Exhibit 13 shows gain or loss in dollars and illustrates the benefits from switching to the put-spreads, i.e. from reducing the drag from vega (note that after a sharp selloff in the underlying index, the volatility tends to rise, resulting in losses if volatility subsequently recedes). The before and after periods below show respective results before and after monetization on March 18th, 2020.

2. Both strategies purchase a single SPX 12/17/21 $2600 put on January 3rd for $113.25 (for more details on monetization please see Bhansali et al. 2020). The active strategy sells this option on March 18th for $581.7 and redeploys this premium by selling 7.26 units of SPX 12/17/21 $1875, and purchasing 7.26 units of SPX 12/17/21 $2100.

Exhibit 13: Decomposition						
	Full Sample		Before Monetization		After Monetization	
	Passive	Active	Passive	Active	Passive	Active
Delta	-103.0	-144.4	195.9	195.9	-298.8	-340.3
Theta	-25.5	-26.6	-15.4	-15.4	-10.1	-11.3
Gamma	132.1	135.0	77.5	77.5	54.7	57.5
Vega	85.8	156.5	210.5	210.5	-124.7	-54.0
Total	89.5	120.4	468.5	468.5	-379.0	-348.0

Source: LongTail Alpha, OptionMetrics. Full sample is Jan 1, 2020 - June 1, 2020.

CONCLUSIONS

We discussed the concept of total return as applied to options and option portfolios. By aggregating the return contribution from the various local derivatives of the options prices ("greeks), we first showed how the total return of options portfolios can be evaluated in terms of the tradeoffs between exposures to the market ("delta"), convexity ("gamma"), time-decay ("theta") and volatility ("vega"), leading to better understanding of the contribution of various drivers to option total returns. Second, by combining options with different strikes and expiries, we can then create portoflios targeting total return from specific underlying drivers. We show via an example how a monetization strategy can be created to target returns from specific drivers. We believe that just as the concept of total return enables investors to create better bond portfolios, the total return concept as applied to options has the potential for creating superior option portfolios and lead to improvement in risk management techniques.

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