Misleading Returns: How Ignoring Cash Flows Can Result in Performance Measurement Errors

This Version: April 15, 2023

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Abstract

We discuss how the traditional approach of measuring performance using time weighted compounded returns can lead to grossly misleading conclusions in the context of two examples of practical interest. First, we show how measuring tail risk hedging performance using only compounded returns, rather than both returns and timing of cash flows in the context of the underlying portfolio, can lead to erroneous conclusions about the value added by such hedges. Second, we show how measuring performance using compounded returns alone and ignoring timing and size of investment flows can result in contradictory conclusions about the long-term profitability of such investments, using the ARKK ETF as an example. We conclude that a more complete approach to performance measurement is essential for investors to not be misled by over-simplified metrics such as compounded returns.

Key Takeaways

- While simple and intuitive, the practice of solely relying on compounded returns only can lead to misleading conclusions
- For example, unlike fully-funded strategies, NAV (Net Asset Value) based returns fails to communicate the efficacy of tail hedges due to their highly volatile return streams. In order to obtain a complete view of a tail hedge strategy, NAV, cash flows, and NAV based returns should be presented together.
- When only using compounded returns, it is also possible to arrive at conflicting conclusions about the profitability of investments if proper attention to fund flows and timing is not included. For example, it is possible for compounded return calculations for a fund to be positive, yet in aggregate investors can suffer total dollar losses.

Performance measurement is a critical part of evaluating the success or failure of an investment strategy. Despite being so central to investing, even amongst professionals there is generally a weak understanding of how different ways of reporting performance can result in different conclusions for the viability of an investment strategy. In practice, time weighted compounded returns are the most common way that funds report their performance. For private equity and other investments such as tail risk hedging, where cash flows matter (a lot), the concept of money weighted returns or internal rate of return, though still not complete, might prove to be more useful. While simple, reliance on one measure alone can result in an incomplete picture of performance, and hence can lead to erroneous investment decisions.

In this paper, cumulative compounded returns are computed by assuming that an investor contributes an amount of capital on day 1 and does nothing else throughout the life of the investment. The initial capital and any gains or losses flow directly into the start of the next period, or are invested from period to period at the internal rate of return r_i for each period i,

$$Compounded\ Return = [(1 + r_1) * (1 + r_2) \dots (1 + r_n)] - 1$$

Typically, fund return streams are provided as a series of per period percentages so investors can simply take the product of their starting capital and the return stream at any point along the series to get an estimate of what their performance may have looked like. Similarly, compounded returns are usually provided in fund documents so investors can easily calculate historical expected performance over a period of time. Though there are standard disclosures that "past performance is not a promise of future returns", investors tend to extrapolate past performance into the future. This standard methodology also allows for easy comparison between funds, such as computing Sharpe ratios, volatilities etc. It is common knowledge that this type of analysis can differ from actual returns experienced by a given investor, but it is assumed that the hypothetical investor who has entered the fund on day 1 has re-invested all cash flows back into the fund and thus this representative investor's experience represents the performance of the fund, so another investor can make an informed analysis relying on this long-term return series.

We hope to demonstrate that this approach, while mathematically correct, can just as often mislead as it can inform. Our solution, discussed in the conclusion, is to supplement numerical performance measurement with a holistic approach that incorporates elements of performance that are not in the compounded return formula. We argue here that paying attention to the cash flows, both their magnitude and timing, as well as the essential purpose of an investment can shed further light on how to paint a composite and useful picture of performance measurement. While mathematically there is nothing wrong with using compounded time weighted returns or IRRs, fixating on only one metric can easily lead to conclusions that don't necessarily lead to wise investment decisions. In other words, it is easy to get lost in the trees of self-consistent and accurate computations while losing sight of the forest of assessing performance.

Case Study 1: Tail Hedging Performance Measurement

Performance measurement and reporting of returns is still an active topic of discussion despite what would seem to be its elementary and unquestionable arithmetic. Indeed, what could be wrong or controversial about computing returns? Is it not as simple as linking together period to period returns using simple compounding and stating the cumulative gain or loss in wealth as a function of the initial investment?

It turns out that, though mathematically this simple recipe for compounding returns is unassailable, there is a lot that it misses, and this still leads to controversy. As recently as April 3, 2023 a Bloomberg news article highlighted the disagreement within the professional investment community on how one manager was reporting returns.[2] In a previous paper (Chang[2022]), we discussed exactly how this kind of confusion can arise when evaluating tail hedges or other insurance type investments.

As an analogy, note that homeowners almost never ask their insurance providers to send them the compounded return statistics on their insurance premiums paid to measure performance. The reason is simple: insurance on homes is bought for its desirable conditional cash flow characteristics, i.e. even though the insurance premium is expected to be a total loss every year, the relatively small insurance premium paid annually protects the home-owner from a catastrophic loss if the house burns down. Computing the total cumulative return on this premium, as is done for traditional investments might mathematically be sound, but conceptually it does not make much sense. This is because people who buy home insurance judge its performance by the *reliability* of the contingent payoffs when they matter.

Compounding returns misses the point that the reason people buy home insurance is because (1) it is cheaper to buy insurance than to set aside a lot of money for a low probability event; (2) if purchased from a reliable party, the insurance pays off when it is needed; and (3) the cash flow is sufficient and satisfactory for the premium cost incurred when an event takes place. None of these driving factors are included in the compounding return computation, which renders the compounding calculation much less useful for such investments. If return computations are an aid to making better future decisions, then this point highlights that return calculations are useless for making better future decisions regarding whether or not to buy insurance if the other, more salient factors are not included.

From the point of view of decision making, note that this performance computation does not help at all. Even if an insurance provider were to come back and report that the cumulative return over the last three decades from buying insurance was -100%, we suspect that despite this dismal cumulative "performance" of our insurance policy over the last three decades, and with the expectation that it will have exactly the same type of dismal performance over the next thirty years, we would still buy insurance for another year. Even though the mathematical expected return on the insurance by itself is a total loss, the primary drivers for purchasing the insurance still hold true.

[2] "Black Swans, Angry Hedge Funds, and How to Make a 3612% Return", Bloomberg News, April 3, 2023.

So what we need to make sense of is the decision to recast the problem with the other dimensions explicitly accounted for. There are three primary reasons that investors hedge (see Bhansali [2014]):

1. Tail hedges deliver marked to market gains during periods of stress.
2. Monetization proceeds come at an opportune time when there is a need for liquidity.
3. Tail hedges improve the overall risk characteristics when combined with a hedged portfolio allowing investors to be more aggressive to achieve higher returns.

A tail hedge overlay is quite similar to our home insurance example. In exchange for a small amount of "premium" spent, the owner of a tail hedge gets protection against a catastrophic market loss. The premium protects over some unknown event over a fixed horizon, without perfect foresight on when such an event might occur. Unlike home insurance, tail hedging in the financial markets has the two other additional features listed above. In a period of crisis in the markets, the tail hedges can be sold ("monetized") and the cash can be put to good use, including re-investing in the markets. The third point above is important because the inclusion of the tail hedge in the portfolio allows investors to build portfolios which have the same or lower downside loss potential as an unhedged portfolio, but also allows them to garner more potential gains.

Having recast the problem with the additional salient dimensions, let us now show the shortcomings of the return computations with and without these additional features accounted for. The strategy here is generically referred to as the "LeftTail Strategy"[3], both as a stand-alone portfolio and a portfolio overlay. In terms of formulas,

$$Return = \frac{PnL}{Starting\ NAV + Subscription}$$

where:

$$Starting\ NAV = Prior\ Ending\ NAV - Redemption$$

$$Ending\ NAV = Starting\ NAV + Subscription + PnL$$

We begin with two scenarios that demonstrate the NAV (Net Asset Value) based return accounting methodology. Exhibits 1-2 differ in the volatility of their respective returns, with Exhibit 2 having returns more akin to a highly leveraged overlay protection strategy, such as a tail hedge. The starting capital for both examples is $10. In all the examples, we will compute two returns. The first return, which we call "compounded return", strings together the returns using the compounding formula. The second return, which we call "dollar return", looks at the total terminal dollar value received, and computes the return of the dollar capital relative to the dollar value initially invested. Note that in both examples, there is no present value factor, since we are computing the ex-post summary return of a time-series of investor experiences.

[3] LeftTail Strategy data is sourced from LTA, OptionMetrics and Bloomberg. Each quarter, the tail hedge strategy spends a quarter of the budget on a new 1Y out of the money tail hedge option on the S&P 500 index. If the current value of any tail hedge exceeds 8x its original purchase price, the position will be fully monetized.

$$Dollar\ Return = \frac{\sum_i PnL_i}{\sum_i Subscription_i}$$

In both Exhibits, the compounded NAV based return equals the actual dollar return of each investment, which should be no surprise:

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	0.4	10.4	0	4.00%
2	10.4	0	0	10.4	0	0.00%
3	10.4	0	0.2	10.6	0	1.92%
		10	0.6		0	
Compounded Return						6.00%
Dollar Return						6.00%

Exhibit 1: Single Subscription Low Return Volatility

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	1	11	0	10.00%
2	11	0	-1	10	0	-9.09%
3	10	0	7	17	0	70.00%
		10	7		0	
Compounded Return						70.00%
Dollar Return						70.00%

Exhibit 2: Single Subscription High Return Volatility

Note that in both Exhibits and respective calculations above, we are inherently assuming the use of a buy and hold strategy, which is what makes it possible to string together single period returns to get long term returns. If there are no additional cash flows in or out of the fund during the life of the investment, the compounded return and dollar return will be equal.

Tail hedges, or insurance contracts in general, are generally not buy and hold strategies. Tail hedges are intended to be time and event specific and proper utilization of tail hedges require active monetization, whether mandatory, following a rules-based approach, or discretionary, as when market tail hedges are sold before expiry through active management. Further, investors in tail hedge funds typically want to access liquidity provided by a monetization event as soon as possible. This can be via a redemption to either offset losses from the underlying portfolio, or for redeployment into the market. Finally, it is possible (and in many cases, likely) for the premium in a tail hedge to decay to zero, which means investors may be required to add a subscription in order to extend and maintain the hedge. For these reasons, we must include cash flows in our analysis when looking at fund performance. From our perspective, ignoring the cash-flows can paint an egregiously inaccurate picture of the value of

tail hedges, which surprisingly is not immediately familiar to many professional practitioners in finance who clearly understand compounding. The reason, as we will show, is that when the cash flows are small compared to the size of the investments, the mismatch between the two measures of return is small. However, when the payoffs and cash flows are large compared to the investment, as in the case of premium-based hedging strategies, the two measures can diverge substantially, to the point of having opposite signs.

Building on our previous examples, we see that when cash flows are included, holding return streams constant, compounded returns and dollar returns are no longer equal. Exhibit 3 and 4 have identical return series to Exhibit 1 and Exhibit 2 respectively, but there is now a redemption in period 1. Considering this new cashflow, the compounded return and the dollar-based return are no longer the same. This divergence can grow quickly as shown in Exhibit 4.

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	0.4	10.4	0.4	4.00%
2	10	0	0	10	0	0.00%
3	10	0	0.19	10.19	0	1.92%
		10	0.59		0.4	
Compounded Return						6.00%
Dollar Return						5.92%

Exhibit 3: Low Return Volatility with Redemption

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	1	11	1	10.00%
2	10	0	-0.91	9.09	0	-9.09%
3	9.09	0	6.36	15.45	0	70.00%
		10	6.45		1	
Compounded Return						70.00%
Dollar Return						64.55%

Exhibit 4: High Return Volatility with Redemption

We can take this analysis one step further, and model returns from funds where redemption or monetization cash flows are of magnitudes more in-line with what an investor would expect to receive from a tail hedge. In Exhibit 5, we see that as the size of percentage returns increase, the difference between the two return calculations continues to diverge. Note, it's broadly recognized in the industry that a 5x or 500% return is not necessarily considered outsized for a tail hedging portfolio. As a matter of fact, and as described in the next section, a 5x to 10x return on premium deployed is quite within expectations for a typical tail hedging strategy during a market event that the tail hedge is targeting.

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	40	50	40	400.00%
2	10	0	0	10	0	0.00%
3	10	0	30	40	0	300.00%
		10	70		40	
Compounded Return						1900.00%
Dollar Return						700.00%

Exhibit 5: Large Tail Hedge Returns with Redemption

Exhibit 6 shows the effects of large negative returns which, again, are expected for any tail hedge portfolio as options decay to zero. Here, the portfolio makes a 5x return in the first period, and the investor redeems the $40 profit. There is no change in value in the second period, and the portfolio loses most of its value in the third period. If we look at the dollar-based return, the investor in this strategy would hypothetically make $31 on a $10 investment. However, because of the effect of the cash flow redemption, the compounded return is -50%. A negative compounded return calculation, when the investor made 4x on the initial investment, is clearly not representative of the investor's true experience in this example.

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	40	50	40	400.00%
2	10	0	0	10	0	0.00%
3	10	0	-9	1	0	-90.00%
		10	31		40	
Compounded Return						-50.00%
Dollar Return						310.00%

Exhibit 6: Large Negative Returns

Finally, in Exhibit 7, we show a return stream where the tail hedge value has decayed to zero in the first period resulting in a -100% return. To continue the tail hedge program, a new subscription of $10 is made in the second period and earns a +500% return, followed by a small loss in the third period. The dollar profit net of total subscriptions is positive in this hypothetical example, but the compounded return is -100%. The first period return of zero effectively corrupted the future return stream since all future returns will be multiplied by the initial -100% return. While the compounding-based computation is not incorrect, it simply does not communicate the positive dollar returns that were realized in this example. While the likelihood of a fund losing its entire value at a single point in time is unlikely in reality, this example is meant to demonstrate that the simple mathematics of NAV based performance calculations may not always be conceptually representative of the actual value to the investor. Anecdotally, in March of 2020, VIX call option strategies and to a close degree S&P 500 index put options strategy demonstrated almost exactly this type of return profile. While these options had lost close to 100% of their premium value over the last ten plus years, they delivered large enough returns to make up for all the cumulative losses in one episode. An observer looking at the cumulative returns even after the large gain would compute the compounded return of such a

strategy to be close to -100%, paying no attention to the gains realized when the hedge was effective. This conclusion would not ascribe any value to the payoffs from the hedge when it was critically valuable to the investor.

Period	Starting NAV	Subscription	PnL	Ending NAV	Redemption	Return
1	0	10	-10	0	0	-100.00%
2	0	10	50	60	0	500.00%
3	60	0	-5	55	0	-8.33%
		20	35		0	
Compounded Return						-100.00%
Dollar Return						175.00%

Exhibit 7: Loss of Premium over a Single Period

As we hope these examples have shown, NAV based returns in isolation are not sufficient to provide a clear picture of performance for tail hedge funds. The inclusion of cash flows and NAV, in the context of the underlying portfolio that is being hedged is necessary; cash-flow magnitude and timing are both important to obtain the correct picture of the tail hedge performance. This is because the NAV and any performance calculation based on it only shows the performance of the non-monetized value remaining in the fund, and the reason tail hedging is implemented is in recognition of the cash flow that can be monetized and possibly extracted contingent on a large market event.

Now that we have established a clear background for appropriate performance computations, we proceed to use this framework to demonstrate how properly including the three main features of why an investor decides to hedge can paint a more complete and less misleading picture.

We present our data in a normalized manner using a simple rules-based passive tail risk hedging strategy. There is nothing special about the example used here, and we simply picked a conservative representative back test from 2017-09-18 to 2021-10-31 where the tail hedge strategy purchases 20% OTM puts and monetizes when the options reach an 8x multiple of their respective premium. Other examples abound, and certainly the reader can take our example and apply it to various hypothetical scenarios to judge the pros and cons of the value of tail hedges, using the proper framework that includes cash flow magnitudes and timing. More details on the experience of actual tail hedge funds through COVID-19 and prior are presented in a recent paper on monetization strategies that use fund data (see Bhansali et. al. [2020]).

To make the computations tractable and transparent, we made the following assumptions in our calculations:

1. Portfolios have a starting value of $100
2. The sum of all subscriptions for the life of the tail hedging strategy are normalized to a $2 annualized “spend”. This means that on average, the cost of the hedge was 2% per year.

3. Month-end returns on the NAV of the tail hedge strategy assume all inflows for premium cash flows occurred at the beginning of the month and all outflows from monetization occurred at the end of the month

Exhibit 8 shows the normalized market value of the tail hedging strategy which is displayed as NAV in blue. The S&P 500 (market) is shown in grey and set to begin at a value of $100. Cash flows are shown as bars where subscriptions are red and redemptions are green.

We observe that the performance of the tail hedge relative to the S&P 500 shows marked to market gains during two periods of market stress where the S&P 500 has the most significant declines. In December of 2018, the S&P 500 declined just under 10% while the return of the tail hedging strategy for the same month was 79.89% (data used for this computation is available in the online supplement). During the COVID-19 crisis of March 2020, the SPX dropped over 12% and the hedging strategy's return was 203.46%.

Subscriptions occur incrementally throughout the life of the strategy as options decay and additional cash is needed to put on new positions. There is a single, but significant monetization event during the COVID-19 crisis when the 8x multiple threshold is reached.

Exhibit 8: LeftTail Strategy Market Value and Cash Flows
Source: LongTail Alpha, OptionMetrics, Bloomberg

When we compound the historical month-end NAV returns of the tail hedge, we get a value of -99.99% suggesting that the tail hedge lost the full amount of total subscriptions in the hedging strategy (please refer to the online supplement). This would seem like dismal performance for a strategy to most investors. However, applying the dollar return methodology paints a very different picture.

When we look at the sum of premium and monetization cash flows in Exhibit 9, we see that the hedging strategy spent a total of $10.00 in subscriptions and earned $5.10 in redemptions for a total net loss of $4.90, or 49%. Although this is still a negative dollar amount, it is much smaller than the -99.99% calculated from the compounded return. Many investors may not mind having a 49% loss on a small amount of premium in order to experience a substantial payoff when the markets are crashing and there is widespread panic and distress. Trying to appraise the value of the strategy solely based on its compounded returns would possibly have distracted attention from the magnitude and timing of the desirable contingent payoff.

	Premium Cash Flow	Monetization Cash Flow
9/18/2017	2.00	
12/14/2017	0.50	
3/15/2018	0.50	
6/14/2018	0.50	
9/20/2018	0.50	
12/21/2018	0.50	
3/14/2019	0.50	
6/20/2019	0.50	
9/19/2019	0.50	
12/19/2019	0.50	
3/16/2020		-5.10
3/19/2020	0.50	
6/18/2020	0.50	
9/17/2020	0.50	
12/18/2020	0.50	
3/18/2021	0.50	
6/17/2021	0.50	
9/16/2021	0.50	
Total	**10.00**	**-5.10**

Exhibit 9: LeftTail Strategy Cash Flows
Note: Assumes an annualized Premium Cash Flow budget of $2 to protect $100.

Combining with the Underlying Portfolio Provides a More Complete Picture of Performance

In this section we demonstrate that the timing and magnitude of the cash flows also plays an important role when combining a tail hedge with the overall portfolio, in terms of increasing long term risk adjusted expected returns.

We will proceed with the same tail hedging strategy, but use it as an overlay alongside a base portfolio solely comprised of the S&P 500 Index. We will also increase the date range of the simulated back test from 1996-01-02 to 2021-10-31 to get a longer-term view. As before, the tail hedge market value and cash flow amounts are normalized so that the annualized sum of all subscriptions is $2 per year. Our total portfolio will have a starting value of $100, and all

subscriptions will be funded from this amount as needed for the tail hedge portfolio. Similarly, any redemptions will be reinvested back into the S&P 500 Index.

Looking at the aforementioned largest S&P 500 Index declines during our sample time period, we can see that the overlay portfolio had a marked improvement over the S&P 500 Index alone. As shown in Exhibit 10, the hedged portfolio relative to the index alone had an improved drawdown of 1.34% in December 2018, and 6.06% in March 2020.

	S&P 500 with LeftTail Strategy	S&P 500
...		
10/31/2018	-6.10%	-6.94%
11/30/2018	1.25%	1.79%
12/31/2018	-7.84%	-9.18%
...		
1/31/2020	-0.21%	-0.16%
2/29/2020	-6.76%	-8.41%
3/31/2020	-6.45%	-12.51%
...		

Exhibit 10: S&P 500 with LeftTail Strategy Overlay
Source: LongTail Alpha

Exhibit 11 shows several total portfolio level statistics for the two strategies. Focusing on the full history of the strategy in the CAGR (Cumulative Annualized Growth Rate) column, we can see that the hedged portfolio underperformed by about 57bp on an annualized basis. However, the hedged portfolio had almost 11% less of a drawdown, an improved left tail skew and a lower volatility. This improvement in the overall risk characteristics, when compared to the index portfolio alone, allows the investor to be more aggressive in order to achieve higher returns.

If the two portfolios were normalized to target the same 15% volatility, the hedged portfolio would outperform by around 80bp on an annualized basis. In other words, for the same amount of risk, the investor is able to generate more long-term returns. This idea has been discussed by us in previous papers on how monetization and re-investment based on even very simple rules can result in substantial long-term increase in risk-adjusted returns (see Bhansali et. al. [2020]). The results of this exercise suggest that tail hedged market exposure is a possible benchmark for liquid equity market exposure when drawdown risk is a concern. While this is an interesting idea, as of this writing the lack of uniformity in underlying tail hedging strategies has made it hard for investors to identify what a proper tail hedged equity benchmark should look like. We believe that over time more investors will see the value of aggregation and improved risk adjusted performance to adopt equity market benchmarks with built in tail hedges, and standardization will likely develop.

	Total Return	CAGR	CAGR 15% Vol	Max Drawdown	Calmar Ratio	Monthly Sharpe	Monthly Vol (ann.)
S&P 500 with LeftTail Strategy	547.08%	7.49%	8.69%	-45.07%	0.17	0.63	12.69%
S&P 500	641.93%	8.06%	7.89%	-56.78%	0.14	0.58	15.21%

	Monthly Skew	Monthly Kurt	Best Day	Worst Day	Best Month	Worst Month
S&P 500 with LeftTail Strategy	-34.00%	0.3	6.77%	-6.84%	10.12%	-12.94%
S&P 500	-61.00%	1.13	11.58%	-11.98%	12.68%	-16.94%

Exhibit 11: Overlay Summary Statistics
Source: LongTail Alpha

Case Study 2: How Timing and Size of Flows Distorts Performance Measurement During Bubbles and Busts: The Case of ARKK ETF

The years 2020 to 2022 saw an incredible flood of money into the financial markets. Immense liquidity was injected by the Federal Reserve and other global central banks, and combined with massive fiscal stimulus checks from the Federal Government and retail trading frenzy, resulted in the explosive growth of meme stocks, bitcoin and its variants, and of course the actively managed ETF named ARKK (ARK Innovation ETF).

The ARK Innovation ETF (ARKK) started trading on October 31st, 2014 (Exhibit 12). Its main strategy is to "invest in equity securities of companies relevant to the theme of disruptive innovation" (Source: Bloomberg). Until March of 2020, the ETF oscillated between 20 dollars a share and 60 dollars a share. During this time, the ETF drew in assets slowly, reaching about 2 billion dollars of assets under management (AUM). However, as COVID-19 took hold of the economy, and central banks and governments inundated the economy with money, the AUM of ARKK exploded upwards, reaching an AUM of over 28 billion dollars in early 2021 with a share price over 150 dollars. Fast forward to the end of 2022, the price of ARKK collapsed to about 30 dollars a share, a price not seen since before COVID. The assets under management in late 2022 dropped to below 6 billion dollars, and investors lost a whopping two-thirds (67 percent) in the fund in 2022. The fund's main holdings, which are published daily as required by regulation for ETFs, are Zoom, Exact Sciences, Tesla, Roku, UiPath, Shopify, Teladoc, Beam, and CRISPR, and these have also suffered similar or worse fates (Exhibit 15).

Exhibit 12: ARKK ETF NAV and Share Price
Source: Bloomberg, LongTail Alpha

This is not the first time that there have been bubbles and busts in investment markets, and certainly will not be the last time. Investor tastes in styles can be fickle and generate both positive and negative price momentum and asset flows. Our computations show that even though investors in aggregate have lost close to ten billion dollars invested in ARKK, the compounded annual returns of the fund since inception is still positive. As discussed in the previous section, traditional methods of measuring performance and drawing conclusions from such metrics need to be taken with a huge grain of salt when the underlying returns are large and volatile. While the computations of traditional measurement are mathematically correct, they can lead to conclusions that frequently don't make common sense when faced with bubbles and busts. The second question that attracted us as researchers and proponents of tail risk hedging to this story is the question of whether investors could have somehow managed the risk of investing in this ETF in a way where they could have participated in the gains, but not felt the brunt of all the losses, thus improving true economic returns.

To wit, if we run a total return analysis of the ARKK fund since its inception to the end of 2022, we would find that for a buy and hold investor, the annual compounded rate of return since inception to the end of 2022 would have been 7.1% (Source: LongTail Alpha calculations using Bloomberg data). This would naively suggest that despite the volatility of the ARKK fund, a representative investor would still have made money since inception. This would, however, be misleading! If we compare this with the actual total dollar profit or loss of the fund since inception, we find a completely different, and quite disturbing story. Investors would have lost a whopping 10 billion dollars investing in the fund since its inception to the end of 2022!

Thus, while compounded returns since inception would indeed arithmetically be positive, the actual returns in dollars would be significantly negative. The careful reader can see why this contradiction exists. The main reason is that the return computation does not pay any attention to

the dollar amount at risk and assumes no interim flows. Returns on 1 dollar are treated the same as returns on a billion dollars. And what happens in a world of a flood of liquidity is that prices drive flows and flows drive prices in a vicious, self-reinforcing circle. Once a stock or ETF becomes a retail favorite, it attracts copy-cat investors, and the more investors that get drawn in, the more the price of both the ETF and the underlying stocks rises, since the underlying stocks that compose the ETF are in limited supply and relatively new. Exhibit 13 shows ARKK shares outstanding over time, as well as cumulative investor gains and losses. We note the rapid rise in ARKK shares outstanding during the run up to the peak share price in early 2021.

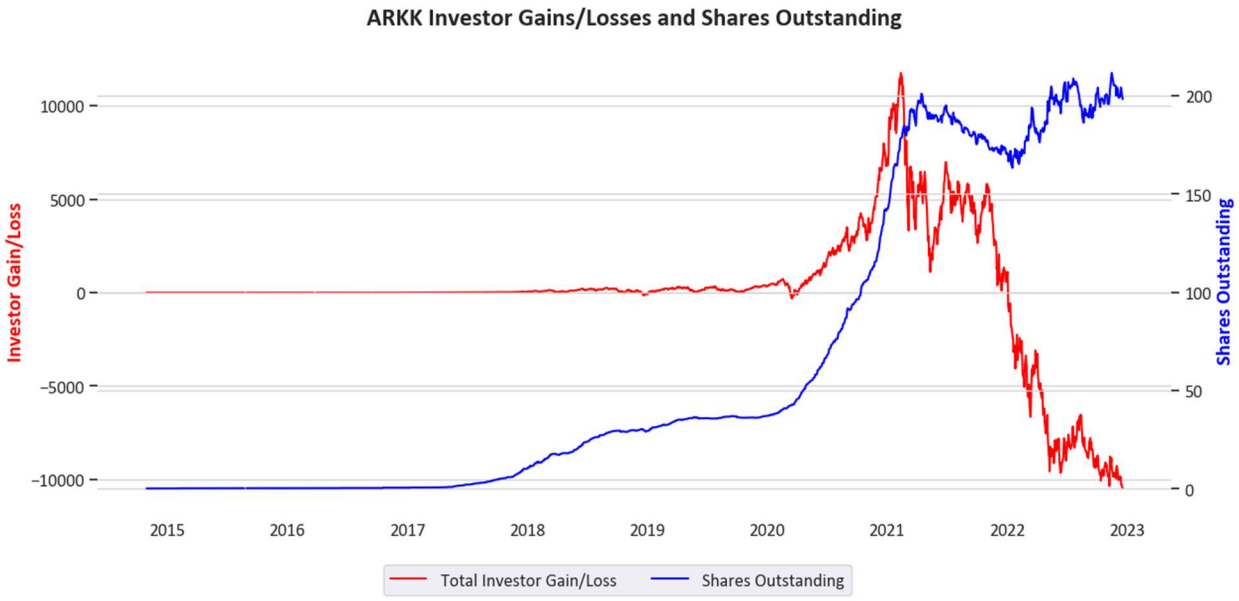

Exhibit 13: ARKK Investor Gains/Losses and Shares Outstanding
Source: Bloomberg, LongTail Alpha

Of course, there is a flip side to the inflow of liquidity. When liquidity evaporates, and investors sell their holdings of the ETF, the act of selling can drive the price of the ETF down rapidly as well. As the ETF liquidates the underlying holdings, which are themselves illiquid, this creates a vicious cycle of further declines in price of the ETF, which results in further liquidation. The problem here is obvious - there is a positive feedback mechanism between price and size of the fund. If the fund is the largest in size when it is most overvalued, collapse and losses can be very likely.

In Exhibit 14 we show the performance of all the ARK funds since inception. The phenomenon of the rate of returns (CAGR) not telling the same story as the dollar gains and losses holds true for all the funds that started in 2014. Again, this can be traced to the fact that in the initial years (2014-2019), the funds had positive returns, which drew in capital. In contrast, the funds that started later (e.g. ARKF and ARKX) have both had negative rates of return and negative dollar returns over their life, because they did not have the opportunity to benefit from the bubble period returns. Since there is significant overlap in the holdings between various ARK funds, it is also feasible that the liquidation of one ARK fund could have adversely impacted the

other funds. Below we show our method for calculating dollar losses, where *shares* denotes the number of shares outstanding, and *t* denotes end of day on day t.

$$losses_t = shares_{t-1} * (price_t - price_{t-1})$$

Description	Ticker	Inception Date	CAGR	Investor Losses ($mm)
ARK Innovation	ARKK	10/31/2014	7.1%	-10490
ARK Next Generation Internet	ARKW	9/30/2014	11.2%	-1580
ARK Genomic Revolution	ARKG	10/31/2014	5.6%	-4280
ARK Autonomous Tech. & Robotics	ARKQ	9/30/2014	10.2%	-620
ARK Fintech Innovation	ARKF	2/4/2019	-8.3%	-1910
ARK Space Exploration & Innovation	ARKX	3/30/2021	-25.6%	-210
The 3D Printing ETF	PRNT	7/19/2016	0.8%	-220
ARK Israel Innovation Technology	IZRL	12/5/2017	-1.9%	-120

Exhibit 14: ARK Invest Funds Returns from inception to end of 2022
Source: Bloomberg, LongTail Alpha

When we take a look under the hood, we find that the component stocks in a fund such as ARKK also suffered a parallel history.

Exhibit 15: ARK Invest Funds Returns
Source: Bloomberg, LongTail Alpha

As ARKK increased in price, the typical stock inside the ETF also increased in price, presumably because of demand from the fund and other investors. As ARKK fell in price, the underlying stocks also sold off aggressively. Part of this could be ascribed to the nature of the

holdings – "new economy", non-dividend paying, "innovation" stocks should be correlated with the "hopes and dreams" of long term speculation. As the ETF is sold, it is forced to sell the underlying stocks to maintain portfolio balance. In other words, the act of risk management via the need for diversification may actually result in overwhelming other considerations. Again, this is not a new phenomenon, and has been observed in the recent past where risk mitigation considerations resulted in the forced feedback driven liquidation of ETFs (such as the inverse vol ETF XIV which imploded spectacularly in 2018).

As an example, take Zoom Video Communications (ZM). The stock price of the company went from about 68 in 2020 to a high of over 450 in late 2020 as the global economy went home, and Zoom became the default technology for essentially all communication during COVID-19. As economies reopened, the stock price dropped back to the pre-COVID-19 levels, erasing all of its gains and losing 85 percent of its value. Bubble and Bust. As a large holder of this stock, ARKK and other pooled vehicles inevitably suffered the same fate.

Hedging Bubbles and Busts

So now that we have demonstrated that compounded return calculations can mislead, what, if anything, could investors have done to actually avoid losing money? The ARKK ETF provides a great laboratory test case for three important questions related to investment decision making and risk management when compounded return computations don't help. First, bubble and bust dynamics of the ARKK ETF shows the impact of flows and positive feedback loops on AUM, price, and its underlying holdings. Second, the bubble and bust dynamic calls into question the concept of measuring performance using compounded returns from inception when returns and flows are large and volatile. As we have shown in this note, a fund can lose billions of dollars in aggregate while still showing positive compounded returns. Bringing in some of the concepts we discussed at the end of the last section, would it be possible for investors to use tail hedging techniques to improve overall investment experience if they hold concentrated positions in bubble stocks?

Thus, in this section, we explore various risk management techniques to evaluate whether their inclusion could have resulted in better economic outcomes for an investor in ARKK. The techniques we explored include systematically de-risking investments in ARKK, hedging using put options on an index like the Nasdaq index, a combination of the last two and some other variations.

Our conclusions are intuitive: when faced with bubbles in idiosyncratic securities such as ARKK that have little robust relationships to broad market indices, it is probably futile to look for options-based strategies on broad market indices. The best strategy, somewhat surprisingly, seems to be to simply de-risk systematically, i.e. sell and rebalance back to cash. Of course, it is possible to data mine the historical data and find some strategy that would have reduced the downside risk, but such fine-tuned strategies are unlikely to be robust in the future. We also did not include put options on the ARKK ETF due to the relative illiquidity of these options.

We discuss the following strategies to estimate the effect of various risk management strategies. Results are displayed in Exhibits 16 to 18.

1. "Buy and Hold" allocates the entire beginning AUM to ARKK shares.

2. "Match Notional" buys put options to cover the beta-scaled notional of the ARKK position. This is implemented using 1 year 20% OTM Nasdaq put options on an annual March cycle. The option position is rebalanced every quarter to match the beta-scaled notional of the ARKK position. The options overlay is funded via a free margin account (i.e. with zero borrowing costs), hence the ARKK share count equals the buy and hold strategy at every moment in time. The beta scaled notional is calculated by multiplying the market value of the ARKK position by the rolling 6-month beta of ARKK against the Nasdaq.

3. "Match Notional, Sell ARKK" strategy funds the option overlay described above by selling ARKK shares. It does not use margin. Therefore, ARKK shareholdings decrease over time, with any options proceeds being put into cash. Options proceeds are never used to buy more ARKK.

4. "Just Sell ARKK" strategy will sell ARKK shares in the same quantities as the "Match Notional, Sell ARKK" strategy above. It puts the sale proceeds into cash. It holds no options positions.

5. "Match Notional, 1.5x Monetization" strategy is similar to the "Match Notional" strategy, but 50% of the option position is sold if the option value rises 50% from the purchase price, with the proceeds being put into cash. The option position is then rebalanced at the beginning of the next quarter to match the beta-scaled notional of the ARKK position at that time; options proceeds are never used to buy more ARKK.

The table in Exhibit 16 shows back-test results for full history of ARKK. The table in Exhibit 17 is for the "bubble" period, i.e. from the inception of the fund on October 31, 2014 to Feb. 12, 2021 when the share price peaked at 155 dollars a share. The table in exhibit 18 is for the "bust" period from Feb. 12, 2021 to the end of December 2022. The first column is dollar return in millions of dollars.

As we see, over the full period of the fund to the end of 2022, the fund cumulatively lost over 10 billion dollars to the end of 2022. The second column shows the compounded annualized growth rate. In the first table, we see that this metric, despite the massive dollar losses, is still positive at 7.1 percent! On a total cumulative return basis, this metric would lead to a total return (non-annualized) of 74 percent for the first dollar invested at the inception of the fund in 2014. The final two columns show the Calmar ratio and the maximum drawdown statistics.

Purchasing a rolling hedge on ARKK using 20%, 1 year out of the money puts on the Nasdaq index does not help reduce the dollar losses by much. This is primarily because the hedges cost premium, and most of the time the hedges do not go in the money due to the basis

risk between the ARKK fund and the Nasdaq. In fact, over the full period, these hedges increase the cumulative dollar losses and also reduce the compounded rates of return. In other words, a broad macro hedge would have been inappropriate for a fund with correlated and idiosyncratic momentum exposure such as ARKK. We also see that the hedging strategy that sells the appropriate number of shares to fund the purchase of Nasdaq index options does not improve performance either. The strategy that dynamically reduces the exposure to the stock improves both the dollar losses and the cumulative return, but does so at the cost of reducing the compounded returns in the bubble period.

In Exhibit 19 we repeat the results for the Nasdaq index ETF QQQ, and find that over the same period, an investor would generally have been better off holding such a broadly diversified index fund, where the weights of many of the individual securities is significantly lower. As a reference, at the end of 2022 the QQQ ETF had a market value of close to 150 billion, i.e. thirty times larger than the ARKK ETF. Again, the simple strategy of buying tail hedges on the QQQ ETF would not have helped the cumulative returns, though episodically it would have indeed assisted in reducing drawdowns.

Strategy	Gain/Loss (mm)	CAGR	Total Return	Calmar	M.D.D.
ARKK Buy and Hold	-10487	7.1%	74%	0.09	-79%
Match Notional	-12149	0.4%	3%	0.00	-87%
Match Notional, sell ARKK	-9740	4.5%	43%	0.06	-80%
Just Sell ARKK	-8294	9.3%	106%	0.13	-73%
Match Notional, Monetize	-11843	2.1%	18%	0.02	-85%

Exhibit 16: Full Period Performance: 10/31/2014 – 12/20/2022
Source: Bloomberg, LongTail Alpha

Strategy	Gain/Loss (mm)	CAGR	Total Return	Calmar	M.D.D.
ARKK Buy and Hold	11276	38%	659%	0.90	-42%
Match Notional	10749	37%	623%	0.89	-41%
Match Notional, sell ARKK	9423	35%	558%	0.88	-40%
Just Sell ARKK	9888	36%	591%	0.88	-41%
Match Notional, Monetize	10766	37%	629%	0.90	-41%

Exhibit 17: "Bubble" Period Performance: 10/31/2014 – 02/12/2021
Source: Bloomberg, LongTail Alpha

Strategy	Gain/Loss (mm)	CAGR	Total Return	Calmar	M.D.D.
ARKK Buy and Hold	-21989	-57%	-79%	-0.72	-79%
Match Notional	-23121	-61%	-83%	-0.74	-83%
Match Notional, sell ARKK	-22424	-58%	-80%	-0.72	-80%
Just Sell ARKK	-21290	-54%	-76%	-0.70	-77%
Match Notional, Monetize	-22817	-60%	-82%	-0.73	-82%

Exhibit 18: "Bust" Period Performance: 02/12/2021 – 12/20/2022
Source: Bloomberg, LongTail Alpha

Strategy	Gain/Loss (mm)	CAGR	Total Return	Calmar	M.D.D.
QQQ Buy and Hold	60271	13%	166%	0.36	-36%
Match Notional	41529	11%	126%	0.26	-40%
Match Notional, sell QQQ	41331	10%	122%	0.28	-37%
Just Sell QQQ	58090	12%	157%	0.38	-32%
Match Notional, Monetize	45971	11%	136%	0.29	-39%

Exhibit 19: QQQ Full Period Performance: 10/31/2014 – 12/20/2022
Source: Bloomberg, LongTail Alpha

Conclusions

By using two current and salient examples we hope to have demonstrated that using only compounded returns for performance measurement has the potential to mislead investors into making the wrong decisions regarding the viability of investments. In the case of using compounded returns to measure the performance of tail risk hedging, the measurement and decision errors arise from not treating tail hedging in its proper context, i.e. as a strategy whose benefits can only be evaluated in combination with the underlying portfolio whose returns it hedges. The error in this case thus emerges from omission of a salient feature of hedging, that it "hedges", and that compounding return calculations that treat the hedge in isolation ignore this feature and may lead to the wrong conclusions. The fix, in this example, is to correct the error by "integrating" the hedge with the underlying portfolio.

In the second example, we demonstrate how measuring compounded returns while ignoring fund flows can also result in conflicting messages about the value of such an investment. By using the retail favorite ARKK ETF we show that since the compounding formula ignores the timing of flows, it leads to positive compounded returns even though investors cumulatively could have suffered billions of dollars of losses over the same period. The remedy here is somewhat different – we find that no simple hedging strategy would have resulted in reducing the dollar losses. Indeed, the solution would have been simply to realize that returns in themselves can be misleading if they don't pay attention to the impact of flows on performance, and indeed the best strategy for managing risk would have been to manage actual financial exposure.

While compounding return calculations are fundamental to performance measurement and hence to investment decision making, we hope to have demonstrated that in isolation these calculations do not paint a complete picture, and hence investors should take a holistic approach to performance measurement and portfolio. There is no doubt in our minds that there are additional examples of the shortcomings of not only compounded return calculations, but indeed of any simple, "one-size-fits all" metrics for performance measurement. Thus, in order not to be misled, investors should pay attention to all the salient features of any investment, and paying attention to the timing and magnitude of cash flows is one critical element that obviously cannot be ignored.

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The pricing source(s) for the data and information used in this paper include pricing provided by independent third-parties. The daily performance presentation is not an official record and should not be treated as such. The data shows the variability of the market value of a tail hedge overlay using intra-month pricing, which may or may not necessarily be captured in official NAV estimates. Unofficial intra-month daily returns are computed by using data from the independent third-parties, and are adjusted by LongTail Alpha to capture intra-month performance fluctuations net of fees. It also contains the history of the strategy's premium cash inflows used to fund the tail hedge overlay as well as its monetization cash outflows. All cash flows are reflective of the strategy's cash flows, but normalized to a $2 per year annualized spend. Examples of the normalized cash flow calculations are available upon request.

Overlay market values are for illustrative, informational purposes only. They are computed using actual fund flows and net of fees performance. All cash flows are reflective of the strategy's cash flows, but normalized to a $2 per year annualized sped. This implied that the total notional value being hedged is $100 with a $2 annual implementation cost.

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Max Drawdown is the worst peak to through return since inception.

The skewness of a dataset measures the degree of distortion from the symmetrical bell curve in a probability distribution and can be calculated by subtracting the mode from the mean and dividing the difference by the standard deviation

Volatility is the standard deviation of returns annualized.

CAGR 15 Vol (Cumulative Annual Growth Rate) represents the return since inception annualized assuming the returns were scaled to achieve a 15% annualized volatility.

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